

INCENTIVE AND MORAL HAZARD IN QUALITY ASSURANCE, PROCUREMENT MANAGEMENT,  
AND HIERARCHICAL CONTROL: AN AGENCY THEORETICAL PERSPECTIVE

BY

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THIS DISSERTATION IS DEDICATED TO MY PARENTS FOR THEIR EVERLASTING LOVE

- MR. TZONG-GUANG YANG AND MRS. FU-GUEY WON YANG

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In this dissertation, hierarchical agency models are proposed to discuss the cooperative nature of departmental interdependence and the interest conflict between the principal and the agents in production environments. There are three functional areas discussed in this study: quality assurance, procurement management and production supervision. These models are established according to the expertise required in the hierarchy. It is the principal's objective to provide an appropriate incentive to reduce the moral hazard problem.

The first model considers the recent trend of the integration of quality and production responsibilities. Therefore, both quality and quantity are contracting attributes. In general, the optimally designed contracts are strictly increasing in quantity and quality produced. The worker is better off with this arrangement, not only through the

increased compensation, but also through the enlarged job responsibilities. The exact incentive scheme depends on the agents' risk attitudes and the quality-enhancing technology.

The second model deals with the interaction between procurement and production management. The production costs crucially depend on the materials, components and subassembly purchased by the procurement department, and on the effort expended by the production department. Linear profit-sharing and target-setting incentive schemes are adopted in this environment. The materials' quality is used to adjust the cost target for the production department. It is shown that in both first-best and second-best cases, profit-sharing compensation is always preferred to fixed salary. The agents' compensations are tied to their positions when the principal lacks a costless monitoring mechanism. Agency cost is then considered as the expected value of getting a perfect monitoring mechanism.

The third model, unlike the first two models with an existing hierarchy, considers the necessity of production supervision and the payoff to establishing an expanded hierarchical structure. The principal's limited span of control and the agents' moral hazard problem explain the desirability of separating the principal from direct production supervision. It is shown that it is indeed to the principal's benefit to expand the hierarchy levels and organization size in many circumstances.

## CHAPTER 1 INTRODUCTION

Increasing global interaction, competitiveness, and the advent of rapid communication in the twentieth century have provoked a revolution in the business environment. The interdependence and cooperation among departments in any one company could well spell the company's success or failure. The employees' motivation, costs and efficiency are among the elements that have significant impact on the departmental relationships and performance, and thus require close management. This study will express the impact each has on the other.

Typically, in a profit/non-profit organization, the largest cost faced by the owner are those for employees' wages, salaries, and benefits. They often absorb half or more of the organization's total revenues (Arnold and Feldman, 1986, p. 340). Employees exchange their time, ability, skills, and effort for valued rewards and personal satisfaction. Therefore it is very important for organizational success that compensation systems are designed in an effective fashion to motivate and maintain the employees' performance. Among various types of compensation systems, Arnold and Feldman (1986, p. 345) indicate that payment schemes have the strongest ability to fulfill the functions of encouraging members to join or stay, rewarding attendance, and improving performance. A recent survey (OR/MS Today, 1990, p. 14) conducted by the Institute of Industrial Engineers to identify issues affecting



productivity and quality in the United States workplace indicates that the primary motivation for employees to increase productivity is financial reward, followed by personal recognition and increased responsibilities and decision-making opportunities. Yet, almost 80 percent of the respondents said that management lacks the commitment to implement productivity programs for employees such as profit-sharing plans and training programs. This is borne out by several studies.

Lawler (1971, p. 158) cites six separate studies of the relationship between payment schemes and performance in the United States, and finds evidence that indicates that payment is not very closely related to performance in many organizations claiming to have merit salary systems. More recently, the Wall Street Journal (July 10, 1990) reports a survey conducted by Brooks International. The survey results indicate that only a quarter of the respondents believe that management does an excellent job of rewarding work groups who make quality improvements. Some companies press employees for quality improvements but base rewards solely on the number of units produced.

An improperly designed compensation system will also affect the employees' performance negatively. For instance, conventional piece-rate schemes for production workers might sacrifice quality for quantity, while portfolio managers paid on the basis of annual accounting profits will sacrifice long-term profitability for short-term earnings. Therefore, a careful examination of alternative compensation systems becomes a necessity for any organization that searches for success.

There are numerous theoretical and empirical studies discussing the issue of designing incentive compensation schemes. One of the most important and popular approaches is Agency Theory. What follows is an overview of that theory, and a statement of the objective and organization of this dissertation.

### An Overview of Agency Theory

The term "agency" has its historic origins in Roman law (Ross, 1974). An agency relationship exists between two or more parties when one of these, designated the principal, seeks to motivate another, the agent, to choose his actions in a way advantageous to the principal. Agency relationships can be widely observed, not only in history but in the contemporary world. Arrow (1985, p. 37) has stated that "the agency relationship is a pervasive fact of economic life . . . analogous interactions are virtually universal in the economy, representing a significant component of almost all transactions." Examples include patient/doctor, shareholder/executive, insurer/insured, manager/workers, client/accountant, and reader/writer. The agency situation or problem is particularly relevant when ownership and control are separate, as is frequently the case in American corporations (Dyl, 1988). Consider this notion of an agency problem in the context of the relationship between stockholders and an executive officer. The stockholders act as principals, delegating the daily operations of the firm to the executive. Stockholders are not in a position to monitor closely the actions of the executive, nor are they as well informed as the executive

as to what are the appropriate actions to take. In an attempt to bring the executive's interests in line with those of the shareholders, the executive is often given some complex compensation package.

This problem is non-trivial because generally the agent's choice of effort level and ability are known only to himself, and there is no immediate incentive for him to truthfully reveal that information to the principal. In addition, because of environmental risk and the agent's typically risk-averse attitude, the agent's behavior involving his assignment is usually not in the principal's best interest. The principal, on the one hand, tries to have the agent share uncertain environmental risk and, on the other hand, must provide enough incentive to attract the agent to the job, or to motivate him to work harder. Thus, agency models incorporate two basic phenomena of organizations: incomplete information and a goals conflict between members of the organization. The trade-off between providing motivation and encouraging risk sharing becomes a general feature of agency problems.

Agency theory views this situation through the design of contracts that maximize the principal's utility and take several factors into account: (a) the relationship between output and the incentive scheme offered; (b) the alternative job opportunities from outside markets; (c) the allocation of risk between the agents and the principal; and (d) the preferences of the principal and the agents with respect to income and nonpecuniary outcomes. The various factors involved and the unavoidable need to deal with stochastic payoffs and costs often make the problem difficult to formulate and to solve. Restrictive results from some basic models do offer rich understanding about incentives and conflicts

within organizations, and provide useful insights in the construction of contracts to guide and influence agency relations in the real world.

In the agency literature, many models come from popular phenomena in the economy. Plentiful examples include insurance policy (Zeckhauser, 1970; Pauly, 1968; Shavell, 1979; Harris and Raviv, 1978; Rothschild and Stiglitz, 1976; Wilson, 1977), auditing and responsibility accounting (Ng and Stoeckenius, 1979; Atkinson, 1978; Holt, 1980; Antle, 1982, 1984; Suh, 1987, 1988; Demski and Sappington, 1989), portfolio selection (Jensen, 1976, 1983; Eaton and Rosen, 1983; Narayanan, 1985; Cohen and Starks, 1988), salesforce compensation (Basu et al., 1985; Nalebuff and Stiglitz, 1984; Coughlan and Sen, 1985; John and Weitz, 1985; Lal, 1986; Lal and Staelin, 1986), government contracting (McAfee and McMillan, 1986, 1987), organization behavior (Williamson et al., 1975; Stiglitz, 1975; Fama, 1980; Fama and Jensen, 1983; Tirole, 1986), national defense contracts (Berhold, 1971; Cummins, 1977), resource allocation (Harris et al., 1982; Harris and Townsend, 1981), and public organization (Hansmann, 1981; Becker and Stigler, 1974; Ross, 1979).

### Objective and Organization of the Dissertation

In his book on "The New Science of Management Decision," Herbert Simon stated a general picture of an organization (1960, p. 40):

An organization can be pictured as a three-layered cake. In the bottom layer, we have the basic work process. In the middle layer, we have the programmed decision-making processes, the processes that govern the day-to-day operation of the manufacturing and distribution system. In the top layer, we have the nonprogrammed decision-making

processes, the processes that are required to design and redesign the entire system, to provide it with its basic goals and objectives.

Hierarchy stands not only for degrees of highness or lowness, for this tends to hide its significance. Each level is an inclusive clustering or combination of interdependent groups, to handle those aspects of coordination that are beyond the scope of any of its components (Thompson, 1967, p. 59). In this dissertation, hierarchical agency models are utilized to discuss the cooperative nature of departmental interdependence and the interest conflict between the principal and the agents in production environments. A perfectly competitive market is assumed for the final products. Three hierarchical agency models are proposed to discuss the agency relationships in quality assurance, in procurement management, and in production supervision, respectively. The ideas developed in this study are partly inspired by the significant improvement in industries through implementing Just-in-Time philosophy in various areas such as the consolidation of production and quality, the vertical integration of procurement and production, and the highly autonomic spirit among employees.

This study provides several interesting insights into how to resolve conflicts of interest within organizations by carefully designed compensation schemes. Unlike conventional contracts, the pattern and design of incentive systems depend on factors such as the agents' attitudes of sharing risk and expending effort, as well as on their production contributions. This study also shows how the organization can be more profitable and efficient by improving several exogenous variables or by adjusting its structure.

A literature review of agency theory is given in chapter 2. An agency model of quality assurance and job enlargement in production management is presented in chapter 3. An agency model considering linear profit-sharing and target-setting compensations in procurement management is presented in chapter 4. An agency model regarding production supervision and moral hazard in hierarchical control is presented in chapter 5. This dissertation concludes with a summary and further extensions of each model in chapter 6.

## CHAPTER 2

### A LITERATURE REVIEW OF AGENCY THEORY

Since the original papers by Wilson (1968), Spence and Zeckhauser (1971), and Ross (1973, 1974), substantial attention has been given to the development of agency theory. Agency theory has been viewed as the neoclassical response to the questions raised by March and Simon (1958) regarding the behavior of an organization of self-interested agents with conflicting goals in a world of incomplete information (Levinthal, 1988). There are basically two types of incomplete information under discussion. The first, referred to as the moral hazard or incentive problem, reflects the inability of the principal to costlessly observe the agent's decision. The second, referred to as the self-selection or adverse-selection problem, reflects the unwillingness of the agent to reveal his private information about the state of nature, his abilities or productivity. Most agency literature has focused on the moral hazard issue, but the more recent work has introduced elements of self-selection (Myerson, 1982; Baron and Myerson, 1982; Demski and Sappington, 1984, 1987, 1988). This chapter focuses on formal, mathematical statements of the agency relationship and largely ignores the less formal stream of positive research by Jensen and Meckling (1976), Jensen (1983), and Fama and Jensen (1983).

In what follows, the basic models and their mathematical justifications are presented. Then, various extensions and related literature are discussed. The chapter concludes with a summary.

### The Basic Agency Model

The basic models dealing with the moral hazard problem and the self-selection problem are presented as follows.

#### The Moral Hazard Problem

The basic agency model focuses on a two-player (the principal and the agent), one-period situation. The agent's effort (or action),  $\alpha$ , together with a random state of nature,  $\theta$ , are assumed to generate outcome  $x$  according to  $x=x(\alpha,\theta)$ . The principal, on the one hand, wants the agent to share risk and, on the other hand, must provide enough incentive to attract the agent to the job, or to motivate him to work harder. The difficulty comes from the fact that the principal can only observe the agent's outcome  $x$ , but not the agent's effort  $\alpha$ , or the random variable  $\theta$ . Given the observed outcome, the principal can only make probability judgments about the agent's effort. It is assumed that both parties are rational and use von Neumann-Morgenstern utility functions in their decision-making processes, where the utility functions are known to both parties. The principal is typically assumed to be less risk averse than the agent (Mirrlees, 1976). Let the principal be risk neutral, and the agent be risk averse with utility function  $H(w,\alpha)$  defined over income ( $w$ ) and effort ( $\alpha$ ). The model



further assumes the utility function is additively separable into a utility for income,  $U(\cdot)$ , and a disutility of effort,  $V(\cdot)$ , i.e.  $H(w, \alpha) = U(w) - V(\alpha)$  with  $U' > 0$  and  $U'' < 0$ . The interpretation is that  $\alpha$  is a productive input with direct disutility for the agent and this creates an inherent difference in objective between the principal and the agent (Holmstrom, 1979); Harris and Raviv (1979) have proved that the moral hazard problem can be avoided when the agent is risk neutral. The agent is effort averse, and his disutility of effort increases at a nondecreasing rate, i.e.  $V' > 0$  and  $V'' \geq 0$ . Mirrlees (1974, 1976) suggests that  $\theta$  be suppressed and views  $x$  as a random variable with a distribution  $F(x; \alpha)$ , parameterized by the agent's action. Given a distribution of  $\theta$ ,  $F(x; \alpha)$  is the distribution induced on  $\alpha$  via the relationship  $x = x(\alpha, \theta)$ . The principal's problem is to design a sharing rule or contract that elicits an appropriate effort level from the agent. The sharing rule or contract,  $S(x)$ , which is constructed on the basis of the principal's observation of the agent's performance, can be generated by the following program:

$$(M) \quad \text{Max} \quad \int [x - S(x)] f(x; \alpha) dx \quad (2.1)$$

Subject to

$$E\{H(S(x), \alpha)\} = \int U(S(x)) f(x; \alpha) dx - V(\alpha) \geq H^0 \quad (2.2)$$

$$\alpha \in \underset{\alpha' \in A}{\text{Argmax}} \quad [\int U(S(x)) f(x; \alpha') dx - V(\alpha')] \quad (2.3)$$

where  $E\{\cdot\}$  denotes the expectation operator, the notation "argmax" denotes the set of arguments that maximize the indicated objective function,  $H^0$  is the agent's market reservation value or opportunity cost, and  $A$  is the set of all possible actions. The first constraint is

the so-called "individual rationality constraint" that reflects the fact that the contract must offer the agent an expected utility that is at least as great as his opportunity cost (utility). The second constraint is the so-called "incentive compatibility constraint" that indicates that the agent chooses the action that maximizes his own utility.

In most of the literature (except Grossman and Hart's (1983) convex programming approach) it is assumed that the second constraint in (P) can be replaced by the first-order condition derived from the agent's expected-utility-maximizing efforts, i.e.

$$\int U(S(x)) f_{\alpha}(x; \alpha) dx - V'(\alpha) = 0, \quad (2.4)$$

where  $f_{\alpha}(\bullet)$  is the derivative of  $f(x; \alpha)$  with respect to  $\alpha$ . Mirrlees (1974) points out that the first-order condition may not in general be necessary for the incentive compatibility constraint, unless the contract elicits a unique action on the part of the agent at the optimum. If there is not a one-to-one correspondence between contract and action, then the first-order condition is not even a necessary condition for the optimality of the contract. Rogerson (1985) suggests the correct treatment of the conditions sufficient for necessity of the first-order condition: notably, if the probability density function of the outcome,  $f(x; \alpha)$ , has the monotone likelihood ratio (MLR) property and the cumulative distribution function (CDF),  $F(x; \alpha)$ , is convex, then the first-order approach is valid. The MLR condition states that more effort ( $\alpha$ ) yields more outcome ( $x$ ), and implies that increases in effort will shift the distribution of  $x$  to the right in the sense of first-order stochastic dominance (SDC) (Milgrom, 1981). By the MLR,  $F(x; \alpha)$  decreases in  $\alpha$ , i.e. the probability of an outcome less than or equal to

some  $x^\circ$  decreases as the agent works harder. The CDF convexity condition requires that this function decreases at a decreasing rate, i.e. the CDF is a form of stochastic diminishing returns to scale (Rogerson, 1985).

Jewitt (1987, 1988) argues that the CDF condition is not fulfilled by most of the distributions commonly occurring in statistics, and Rogerson's conditions cannot be directly adapted to the multidimensional case. Jewitt further suggests an over-sufficient but more easily verified total positive (TP) density function, along with a specified utility function as the justification. A function  $f(x;\alpha)$  is said to be total positive of degree  $n$  ( $TP_n$ ) if for each  $x_1 < x_2 < \dots < x_n$ ;  $\alpha_1 < \alpha_2 < \dots < \alpha_n$ , we have

$$\begin{vmatrix} f(x_1, \alpha_1) & \dots & f(x_1, \alpha_k) \\ \vdots & & \vdots \\ \vdots & & \vdots \\ f(x_k, \alpha_1) & \dots & f(x_k, \alpha_k) \end{vmatrix} \geq 0 \quad \text{for } k=1, 2, \dots, n.$$

If  $f(x;\alpha)$  is TP for all  $n$ , then it is said to be totally positive (TP). Jewitt indicates that any exponential family (including normal, exponential, Poisson, binomial, etc.) is totally positive in an appropriate parameterization. The utility condition can be satisfied by any constant absolute risk-averse utility function and any nondecreasing relative risk-averse utility function with an Arrow-Pratt coefficient of relative risk aversion bounded above one half. In order to avoid problems of existence of an optimal sharing rule, bounds are placed on the allowable payments to the agent, and the set of possible sharing rules is assumed to be compact. The bounds on possible payments may be

justified on the basis of institutional features such as limited liability and bankruptcy.

Given that the first-order condition holds, determination of the contract  $S(x)$  can be treated as an isoperimetric optimal control problem (Intriligator, 1983, p. 318). Letting  $\lambda$  and  $\mu$  be multipliers on (2.2) and (2.4), respectively, the Hamiltonian is

$$L = \int \left\{ [x - S(x)] + \lambda [U(S(x)) - V(\alpha) - H^0] + \mu [U(S(x))f_\alpha(x; \alpha)/f(x; \alpha) - V'(\alpha)] \right\} f(x; \alpha) dx. \quad (2.5)$$

Since  $S'(x)$  does not appear in  $L$  explicitly, when the optimal  $S(x)$  is in the interior of the set of feasible contracts, it can be characterized as the solution to the necessary Euler condition, i.e.  $\partial H / \partial S(x) = 0$ .

Thus, the optimal sharing rule can be characterized by

$$\frac{1}{U'(S(x))} = \lambda + \mu \frac{f_\alpha(x; \alpha)}{f(x; \alpha)}. \quad (2.6)$$

The above approach is called "pointwise optimization" in the agency literature.

There are two solution concepts regarding program (M). The solution of (2.1) subject to (2.2) alone is referred to as the first-best solution, which entails Pareto-optimal risk sharing. The solution of (2.1) subject to (2.2) and (2.4) is referred to as the second-best solution, which entails suboptimal risk sharing.

The first-best solution. From Borch's (1962) work,  $S(x)$  is Pareto-optimal from a risk-sharing point of view only if the right-hand side in (2.6) is a constant so that the agent is paid a fixed wage independent of the outcome and the risk-neutral principal bears all

risk. The first-best contract is analogous to a wage contract. The second term of the right-hand side in (2.6) is not a constant by the fact that  $\mu > 0$  (Holmstrom, 1979; Jewitt, 1988). Therefore, optimal risk sharing can be achieved by dropping the (2.4) constraint. Since  $\lambda > 0$ , constraint (2.2) is binding, which implies that the principal can infer the agent's action.

The first-best solution can also be achieved if the agent is risk neutral (Harris and Raviv, 1979). In this case, the principal receives a fixed payment and the agent receives all the residual outcome (Shavell, 1979). Alternatively, if the principal can discover the state of nature or monitor the agent's effort directly, the risk associated with an agency relationship can also be eliminated (Wilson, 1968; Ross, 1973, 1974; Mandlker and Raviv, 1977). Finally, there are asymptotic results regarding the attainment of the first-best solution as the "efficiency" of the agent's effort tends either towards zero or infinity (Shavell, 1979; Grossman and Hart, 1983).

Let the index of the productivity of the agent's effort be denoted as  $\delta$ , with the probability density of outcomes becoming  $f(x|\delta\alpha)$ . If  $\delta=0$ , the distribution of outcomes is not affected by the agent's effort and the problem is merely one of risk sharing. If  $\delta \rightarrow \infty$ , the difference between the second-best solution and the first-best solution tends to be zero (Levinthal, 1988). But, these conditions are quite extreme.

The second-best solution. The shadow price  $\mu$  in (2.6) can be proved to be positive by first-order stochastic dominance (Proposition 1, Holmstrom, 1979) or by the fact that the covariance of  $U(S(x))$  and  $1/U'(S(x))$  is nonnegative (Lemma 1, Jewitt, 1988). Thus, (2.6) shows

that the second-best contract is larger (less) than the first-best contract when the marginal return from effort is positive (negative) to the agent. The second-best contract imposes risk on the agent. The agent bears responsibility over which he has no control. It is "second best" because it sacrifices some risk-sharing benefits in order to trigger a proper effort level. From the principal's viewpoint, the second-best solution is strictly inferior to a first-best solution. The ratio of  $|f_\alpha|/f$  may be interpreted as a benefit-cost ratio for deviation from optimal risk sharing (Holmstrom, 1979). In general, the solution to the agency model is one that is second best.

### The Self-Selection Problem

The self-selection problem arises when before entering into a relationship both parties have different beliefs concerning the exogenous uncertainty. In most circumstances, the agent's information regarding the uncertain state of nature (such as the agent's ability or knowledge of the state of nature) is superior to the principal's. The principal must design some mechanisms (i.e. set of rules) in order to evaluate prospective agents. Salop and Salop (1976) indicate two commonly-used methods: a screening device and a self-selection device. A screening device takes some set of observable characteristics (e.g. past work experience, educational records, etc.), and ranks an applicant's prospective job performance on the basis of his endowment of these characteristics. A self-selection device is a contracting scheme that causes the applicant to reveal truthful information about himself

by his market behavior. In the agency literature, it is the self-selection device that is under discussion.

The following example considers the case in which the agent has private information about his productivity states, while the principal is unable to enforce the full-information solution. It is modified from Demski and Sappington's (1984) and Demski, Sappington, and Spiller's (1988) models. Consider a risk-neutral principal who owns one productive technology that requires as an input the effort,  $\alpha$ , of an agent. The agent's effort, together with the realization of a random productivity factor  $\theta$ , determines the output,  $x$ , produced according to the known relationship  $x=x(\alpha, \theta)$ . The random productivity factor is assumed to be binary, with  $\theta_l < \theta_h$ . The higher  $\theta$  realization places the agent in a more productive setting, i.e.  $x(\alpha, \theta_h) > x(\alpha, \theta_l)$  for any  $\alpha > 0$ . ( $x(0, \theta) = 0$  for all  $\theta$ .) The agent is risk averse with a von Neumann-Morgenstern utility function that is additively separable in monetary reward,  $R$ , and effort,  $\alpha$ . Specifically,  $H(R, \alpha) = U(R) - V(\alpha)$  where  $U' > 0$ ,  $U'' < 0$ ,  $V' > 0$  and  $V'' \geq 0$ . If the agent acquires perfect productivity information before a contract is signed, then his effort choice is equivalent to a choice of output level. Thus, the agent's utility function can be defined over  $\theta$  and  $x$  as  $H(R, x; \theta) = U(R) - D(x, \theta)$  where  $D(x, \theta)$  is the disutility incurred by the agent when he produces  $x$  in state  $\theta$ . It is assumed that  $D(x, \theta_h) < D(x, \theta_l) \forall x > 0$ , and  $D_x(x, \theta) > 0$  and  $D_{xx}(x, \theta) \geq 0 \forall x > 0$  and  $\forall \theta$ , where subscripts denote partial derivatives. Finally, it is also assumed that the marginal disutility of effort for the agent is smaller in a more productive state, i.e.  $D_x(x, \theta_h) < D_x(x, \theta_l) \forall x > 0$ .

Since the principal cannot force the agent to provide truthful productivity information, he must design the choice mechanism that does not give an incentive for dishonesty. It is proved by the Revelation Principle (Dasgupta et al., 1979; Myerson, 1979) that for any equivalence class of any choice mechanism, with no loss of generality, there is an equivalent mechanism that induces truthful revelation of  $\theta$ . Therefore, the principal need only consider the mechanisms in which each agent is constrained to truthfully reveal his productivity parameter as a Nash response. The principal's optimal strategy can be described by the solution to the following program:

(SS)

$$\begin{aligned} \text{Max}_{R_i, x_i} \quad & \sum_{i=1}^h p_i |x_i - R_i| \end{aligned} \quad (2.7)$$

Subject to

$$U(R_i) - D(x_i, \theta_i) \geq H^0, \quad i=1, h, \quad (2.8)$$

$$U(R_i) - D(x_i, \theta_i) \geq U(R_j) - D(x_j, \theta_i), \quad i, j=1, h, \quad (2.9)$$

where  $p_i$  = probability that  $\theta = \theta_i$ ,  $i=1, h$ ;  $x_i$  = the output that the agent will produce when  $\theta = \theta_i$ , in return for reward  $R_i$ ,  $i=1, h$ ; and  $H^0$  is the market opportunity cost (utility).

The individual rationality constraints (2.8) guarantee that the agent will receive at least his reservation utility level if he truthfully reveals his private productivity information. The self-selection or truth-telling constraints (2.9) ensure that the agent will prefer to tell the truth rather than lie about his actual productivity. Demski et al. (1988) have shown that the agent receives his reservation level of expected utility when he observes the low productivity state.



The agent earns strictly positive rents if the high productivity is realized, and the agent is induced to produce the most output in the high productivity state.

### The Extensions

Numerous extensions of the basic agency model have been developed. I classify them into the following categories and discuss them in order: the monitoring model, the multi-agent model, the multi-level model, the dynamic model and the self-selection model.

#### The Monitoring Model

Incorporating monitoring arrangements is a natural extension of the basic agency model. Since agency costs come from the unobservability of the agent's effort, any extra information about the agent's effort will lead to a Pareto improvement toward the first-best solution (Harris and Raviv, 1979). This is true even if the monitoring information is imperfect. Holmstrom (1979) indicates a necessary and sufficient condition for imperfect monitoring information to be of value. Let  $y$  be the monitoring signal. Holmstrom proves that  $y$  is of value if and only if  $[f_{\alpha}(x,y;\alpha)/f(x,y;\alpha)] = h(x;\alpha)$  is false. When the contract is contingent on an additional informative measure, the agent will bear less risk associated with the state of nature. Shavell (1979), however, shows that the value of information goes to zero when the efficiency of the agent's effort goes to zero or infinity. Harris and Raviv (1979) have characterized the form of the optimal monitoring

contract. The latter adheres to the following rule: if the agent's effort is judged acceptable on the basis of the monitoring information, the agent is paid according to a prespecified fee schedule; otherwise, he receives a less-preferred fixed payment. Demski and Feltham (1978) derive the same type of contract (called a budget-based contract), except that they divide performance into favorable or unfavorable on the basis of the outcome.

Baiman and Demski (1980) turn their attention to "investigation strategies" as to when to undertake costly monitoring. Their results include the Pareto optimality of "one-tailed" investigations, wherein either very large or very small variances of costs (but not both) should be investigated by the principal. One of the sufficient conditions for their result is that the agent has a utility function that belongs to the hyperbolic absolute risk-aversion (HARA) class. Young (1986) derives conditions under which "two-tailed" investigations are optimal, assuming non-HARA utility functions for the agent. In a different setting, Dye (1986) proposes a "lower-tailed" investigation strategy that specifies a cutoff value such that an outcome below that cutoff value will trigger an investigation.

Dyl (1988) conducts a regression analysis to examine the effect of monitoring activities on managerial compensation. The results support his hypothesis that in closely-held companies major shareholders engage in monitoring activities that reduce the residual loss portion (due to excessive levels of compensation) of agency costs. The natural logarithm of the percentage of the firm owned by the five largest stockholders is employed as the measure of the degree to which the firm

is closely held. Data regarding the total remuneration of the CEOs of 271 major industrial corporations listed in the Fortune 500 shows that an increase of one unit in the natural logarithm of corporate control is associated with a 9.75 percent reduction in the level of top-management compensation.

Other works on investigation policies include Townsend (1979), Evans (1980), Kanodia (1985) and Lambert (1985).

### The Multi-Agent Model

If all agents face similar states of nature, then comparing performance across agents can mitigate the common risk associated with the outcomes (Holmstrom, 1982; Lazear and Rosen, 1981; Green and Stokey, 1983; Nalebuff and Stiglitz, 1983, 1984). Research examining the risk-sharing benefits of multi-agent contracts has focused on the comparison of the efficiency of individual contracts, which reward each agent for his own outcomes, with tournaments under which reward is a function of the rank order of performance relative to other agents (Levinthal, 1988).

Holmstrom (1982) argues that the rank order of agents is not a sufficient statistic for individual output except in special circumstances as discussed in Lazear and Rosen (1981). Rank-order tournaments may be informationally quite wasteful if outcomes can be measured cardinally rather than ordinally. Holmstrom further suggests that the agent should be evaluated relative to the average performance of all agents. As the number of agents increases, the effect of the common disturbance term can be decreased. His conclusion, however, is

restricted to the additive or multiplicative technology assumption, i.e.  $x_i(\alpha_i, \theta_i) = \alpha + \theta_i$  or  $x_i(\alpha_i, \theta_i) = \alpha(\theta_i)$   $i=1, 2, \dots, n$ , with  $\theta_i = (\eta, \epsilon_i)$  where  $\eta$  is a common uncertainty parameter and  $\epsilon_i$ 's are idiosyncratic risks.

Green and Stokey (1983) consider a model with a risk-neutral principal and a group of homogenous, risk-averse agents. They conclude that the tournament dominates an optimal individual contract if the common environmental risk is large. Nalebuff and Stiglitz (1984) consider the case where the outcome is assumed to be linear in effort. They find that the use of a tournament as an incentive device can induce agents to abandon their natural risk aversion and supply more effort. They also find that the first-best optimum can be approached when there is a large number of contestants, and a penalty to the lowest-ranked individual will be superior to a prize to the highest-ranked individual in motivating effort.

### The Multi-Level Model

Another topic to which agency ideas can be usefully extended is the theory of hierarchy. The study of hierarchical organization was first mentioned by Simon (1957), Williamson (1967) and Lydall (1968), but was not precise enough to generate testable hypotheses.

Mirrlees (1976) is the first to extend the basic agency model to a multi-level structure. In his model, other than the output, the agent's observed performance is also assumed to be uncertain. The accuracy of the observation depends upon the time devoted by the principal to make the observation. Thus, the principal has to decide on the time spent in observation, as well as on the payment function. Production is carried

out by the bottom-line workers only. Each level of the rest of the hierarchy decides the monitoring time and the payment function for its next-level subordinates. Unfortunately, Mirrlees does not provide any well-founded conclusion.

Antle (1982, 1984) considers the owner-manager-auditor relationship and examines the game-theoretic foundations of such an expanded agency model. The auditor's primary role is to produce stewardship information, i.e. information used by an owner and a manager for contracting purposes. Modeling the auditor as a strategic player introduces two complexities. First, the naive mathematical extension of the basic model may yield unreasonable solutions. Second, the nontrivial nature of the subgames implies that randomized strategies by the auditor and the manager may be of crucial importance. These difficulties are illustrated by many examples (Antle, 1982). If the auditor and manager are assumed to play pure strategies, however, inducing truthful reporting by the manager and auditor is optimal. Later, Antle (1984) restricts attention to pure strategies and provides a definition of auditor independence which is a controversial topic in the accounting literature. Basically, Antle assumes that an independent auditor, in choosing among several subgame equilibria in which he receives the same expected utility, will select the one that the owner most prefers, regardless of the effects on the manager. The implications of auditor independence for the optimal compensation scheme for the auditor are discussed via a series of examples.

Baiman et al. (1987) also analyze the effects of adding an auditor to the original principal-agent problem. When the original optimal

contract is second best, they establish conditions that are sufficient to ensure the value of hiring a utility-maximizing auditor. Their model differs with Antle's model in three respects. First, there is no moral hazard problem for the manager. Second, the auditor observes the report submitted by the manager before conducting his audit. Third, the principal cannot observe the outcome and only receives the transferred amount of payment, which is independent of the outcome, from the manager. The results of Baiman et al. (1987) are restricted by the uniform probability distribution assumption over outcomes.

Demski and Sappington (1987) consider a regulatory control model with three individuals: a consumer, a regulator, and a firm. The consumer (acting as the principal) wants to motivate the regulated firm to reduce cost and also motivate the regulator to acquire information that is valuable in directing the firm's activities. Only the firm can expend effort designed to reduce production cost. The regulator's role is to gather information about the firm's cost-reducing ability. Demski and Sappington conclude that the regulator is rewarded if the firm's cost result is consistent with his claim, and is penalized the maximum amount otherwise.

Tirole (1986) considers the coalitions in a three-tier principal/supervisor/agent model. The agent's unobservable production effort, together with an exogenous productivity shock, affects the principal's profit. The supervisor's role is to obtain more information about the agent's activity, and his supervisory effort is assumed exogenous. Thus, there is no moral hazard problem for the supervisor. Under the supervisor/agent coalition, Tirole shows that the constraint

that induces the supervisor to reveal that the state of productivity is low is not binding, i.e. the supervisor acts as an advocate for the agent. Tirole also shows that even if the agent can produce verifiable information himself, most likely there is still scope for a supervisory function.

Suh (1987, 1988) considers a sequential department setting that consists of an intermediate-product division and a final-product division. Suh shows that the optimal allocation of the noncontrollable cost is inconsistent with the concept of responsibility accounting which distinguishes and rewards components of the outcomes for which each division is responsible. One of the reasons is the existence of collusion between divisions. One division could collude with another to affect an apparently noncontrollable cost indirectly. Other reasons are the technological dependence and asymmetric information of the production environment. Thus, basing the final-product division manager's evaluation on noncontrollable intermediate-product costs can serve as an alternative device to costly monitoring or communication.

### The Dynamic Model

Radner (1981) and Becker and Stigler (1974) have shown that dynamic treatments of the agency model induce more efficient results that may achieve the first-order solution asymptotically. Holmstrom (1979) suggests that when the agency relationship repeats itself over time, the effects of uncertainty tend to be reduced, and dysfunctional behavior is more accurately revealed, thus alleviating the problem of moral hazard. As the number of periods increases, the variance in the

average outcomes decreases and more accurately reflects the agent's effort.

Radner (1980) and Rubinstein and Yaari (1980) examine infinite-horizon models in which the same single-period situation is repeated over time, and there is no utility discount over time. Radner (1981) once again considered the same model but repeated a finite number of times. He introduces the notion of an epsilon equilibrium as a means of achieving the first-best outcome in a finite repeated relationship. An epsilon equilibrium is a set of strategies such that each player's average expected utility is within epsilon of being a best response to the other players' strategies.

Lambert (1983) allows the principal and the agent to discount their utilities and assumes the production functions to be separable over time. He derived the optimal long-term contract and found that the agent's compensation depends both on his current and past performance, whether he precommits to the long-term contract or not. Murphy (1986) extends Lambert's results to an incentive-based theory of executive earnings dynamics, both theoretically and empirically. He compares two not mutually exclusive hypotheses: incentives and learning. The incentive model implies that the earning-performance relation and the variance of individual earnings increase with experience, while the learning model implies that the earning-performance relation is strongest during early periods and the variance of individual earnings declines with experience. Both hypotheses predict upward-sloping experience-earnings profiles and a positive relation between compensation and performance.



Harris and Holmstrom (1982) provide new predictions for the behavior of aggregate wages over time. Both the principal and the agents are assumed to be uncertain about agents' abilities, but by a normal learning process, they gradually find out agents' abilities. Harris and Holmstrom conclude that the optimal long-term contract entails a downward rigid wage (i.e. under this contract one's wage never falls over time). The wage is not fully rigid, because the threat of quitting will force the wage to be bid up whenever the market wage is higher than the current wage. The market wage is found to be a function of the worker's current mean perceived productivity minus a term that depends on his age and the precision of beliefs about his productivity. Thus, senior workers earn more on average because they have had more time to have their wages bid up by the market.

Narayanan (1985) borrows Harris and Holmstrom's (1982) model to analyze the portfolio manager's decision in financial policies under asymmetric information. The manager may have incentives to make decisions that result in short-term gain at the expense of the shareholders' welfare, even if long-term contracts are offered. Narayanan shows that this manager's decision is related to the manager's experience, length of his contract, and the risk of the projects. The more experienced the manager is or the longer the duration of the manager's contract, the lower is the probability that the manager would opt for short-term profits. When the projects are riskier, only a smaller proportion of the deviations from the expected cash flows will be attributed to the manager's ability, and this reduces his incentives

for sacrificing the long-term interests of the shareholders for short-term profits.

Holmstrom (1983), based on Azariadis' (1975) work, has explored the dynamics of a contractual labor market in an equilibrium setting. The model assumes homogeneous risk-averse agents, and quits and layoffs are allowed at the beginning of second period. He shows that long-term contracts emerge both when labor is specified and cannot move in the second period and when mobility is costless. Wages are downward rigid but not fully rigid. Holmstrom also points out the seniority rules: different generations of newcomers may have different incomes; especially, younger generations generally earn less, not because of productivity differences but because they enter later. Therefore, in Holmstrom's model, contracts create biases against young members.

### The Self-Selection Model

Salop and Salop (1976) assume that agents differ exogenously only in their probabilities of quitting. The principal's objective is to minimize the turnover costs in a perfectly competitive labor market. This is equivalent to identifying the slow quitters among the applicants and hiring them. Salop and Salop suggest a two-part wage (TPW) scheme as a self-selection device. The TPW would operate in the following manner. The new employee pays the firm an entrance fee of  $D_1$ , in return for which he receives the market average wage,  $W^*$ , and some fixed amount,  $D_2$ . The principal sets  $D_2$  and  $D_1$  such that slow quitters prefer this structure while fast quitters prefer the flat wage structure of other firms. Finally, in equilibrium the agents pay their own training

costs and receive the full value of their marginal revenue product as wages.

Rothchild and Stiglitz (1976), and Stiglitz (1977) examine the equilibrium under asymmetric information in the competitive insurance market and the monopolistic insurance market, respectively. Rothchild and Stiglitz (1976) show that even in a competitive market, a small amount of asymmetric information will exclude a single-price equilibrium. High-risk individuals purchase complete insurance, while the low-risk group purchases partial insurance. Market equilibrium, when it exists, consists of contracts that specify both prices and quantities. But under many conditions, equilibrium may not exist. In a monopoly with imperfect information and nonlinear pricing (which charges customers an amount proportional to the quantity consumed), Stiglitz (1977) proves that the same contract will never be purchased by both high- and low-risk individuals. But, the low-risk individual may not purchase any insurance at all. The high-risk individual, however, always purchases complete insurance.

Harris and Townsend (1981) and Harris, Kriebel, and Raviv (1982) consider intrafirm resource allocation under asymmetric information. Harris and Townsend (1981) prove that the equilibrium parameter-contingent (P.C.) allocations of any mechanism must satisfy certain self-selection properties and such P.C. allocation can be achieved under some mechanism (i.e. some set of rules). Harris, Kriebel, and Raviv (1982) further point out that one of the efficient mechanisms is the transfer pricing scheme. Under certain constraints, headquarters announce a schedule of transfer prices, and each division is asked to

choose a transfer price among them. By the choice of a transfer price, all of the information required for an optimal allocation will be revealed.

Lal (1986) addresses the issue of delegating pricing responsibility to the salesperson. He shows that delegating pricing responsibility to the salesperson is as profitable as centralization when both the salesperson and the sales manager have identical information about the selling environment. But delegation may be more profitable when the salesperson's information is superior to that of the sales manager. Lal and Staelin (1986) follow the same framework but relax the assumption of salesforce homogeneity. They present the conditions under which it may be advantageous for a profit-maximizing sales manager to offer his salespersons the opportunity to choose from a menu of compensation plans. Nevertheless, they are not able to provide the specific design of optimal contracts.

McAfee and McMillan (1986, 1987) introduce a principal-agent framework to the service market. Potential agents, risk neutral and heterogeneous in ability, compete with each other for the contract with the principal. The principal, without knowing the agents' types, designs a contract that exploits the competition among the potential agents and induces them to reveal their types. In this setting, the contract trades off adverse selection against moral hazard. Surprisingly, McAfee and McMillan proved that the optimal contract is linear in the observed outcome in a broad range of circumstances.

### Summary

This chapter introduced the basic models of agency theory and its numerous extensions. Agency theory appears to be a promising research area and there are various avenues of enrichment that will result in valuable propositions (MacDonald, 1984). It has been developed markedly over last 20 years and the impact of this research has been significant (Levinthal, 1988). This study intends to apply agency theory to a hierarchical production environment. This literature review constitutes the basis for the suggested models in this study.

### CHAPTER 3

#### QUALITY ASSURANCE AND JOB ENLARGEMENT IN PRODUCTION MANAGEMENT

Prior to World War I there was very little separation between responsibility for production and responsibility for product quality in the United States. There was rarely a separate functional department for quality distinct from that for production. Between World War I and World War II the concept of "central inspection" began to form (Juran, 1974, p. 7-6). By 1940, the distinction between the functions of quality control and production became solidified (Lubben, 1988, p. 68). By the 1950s, the importance of preventing defects from occurring began to generate serious consideration as a result of the increasing pressure to meet pressing delivery schedules and to maintain higher levels of product quality and reliability. Hence, the quality engineering and reliability engineering functions were introduced. Since then, the separation of responsibilities between quality and production have become more pronounced. By the late 1970s, worldwide competition imposed much greater pressure upon U.S. manufacturing. The quality of Japanese semiconductors and automobiles had reached a much higher level, for example. The term "parts per million" instead of "parts per hundred" emerged as the accepted defect level in electronic components (Lubben, 1988, p. 69).

The trend of the 1980s reverses the process, separating the interrelated functions of quality and production. Almost every book on quality mentions the importance of the production department's responsibility for product quality. Management realizes that as long as there is a separation between the product and the quality of that product, there will remain a lack of commitment, motivation and performance in a manufacturing or service system. Thus, the success of modern industry depends on the full integration of the control of quality and production. But not all quality-related issues are a direct consequence of the production process. Some issues stem from design specifications and supporting guidelines, and others from procurement problems. Therefore, quality control is a responsibility that should be shared by the whole company.

One of the significant concepts of Just-in-Time (JIT) is to return the quality-enhancing function to where the products are produced. Quality assurance is then assigned a more productive role in supporting the manufacturing system. Quality assurance should guide, support and monitor the production process, and develop support programs such as reliability, failure analysis, data analysis, information feedback, and continuing education. The real control of quality is not in the hands of a quality assurance department; rather it is in the hands of the bottom-line production department. From the production employees' viewpoint, the new arrangements will enhance their interest and involvement, and create a break from their previous task-specific jobs. That is, this new responsibility serves as a job enlargement factor for production employees.

Recent trends in the manufacturing concept make the introduction of more appropriate compensation systems a necessity. The old piecework scheme based on production quantity does not provide incentives for quality improvement. The nature of Japanese compensation systems attaches more weight to factors such as effort, cooperativeness, conscientiousness, and the display of initiatives. In fact, many experts suggest that the development of a highly-motivated workforce, and not just a highly-skilled workforce, is the most exacting challenge facing Western managers attempting to establish a JIT production system.

In this chapter, a hierarchical model is proposed to simulate the integration of quality assurance and the production process. Most multi-level agency models consider the added level simply as the source of monitoring information. The added agent provides no productive action and hence cannot affect the realized outcome. In this model, the middle-level quality control manager, in contrast, does have a direct influence on the realized outcome. In what follows, the model is described and the characterization of optimal contracts is then specified. Comparative statics are then analyzed for an additive quality-enhancing technology under certain assumptions. This chapter concludes with a summary of the findings.

### The Model

Suppose a firm consists of three individuals: a risk-neutral, profit-maximizing principal, and two risk-averse, utility-maximizing agents. The two agents are the manager of the quality control



department, and the worker in the production department. The quality control manager is responsible for establishing quality policies and for monitoring the production process. The worker is responsible for producing products and for implementing the quality policies established by the manager. Different expertise is the reason to set up this three-level hierarchy. Production quantity ( $x$ ) is influenced by the production effort ( $\alpha_1$ ) expended by the worker, and a random environmental factor ( $\theta$ ) such as material shortage or machine breakdowns. The quality level ( $y$ ) of the final products, measured by the rate of defectives or some other classification method, is influenced by the manager's guidance effort ( $\beta$ ), the worker's quality-enhancing effort ( $\alpha_2$ ), and an uncontrollable state of nature ( $\epsilon$ ) such as insufficient information of product nature or technology. Although the quality standard is a joint outcome of the efforts of manager and worker, collusion between agents by having a side contract (Tirole, 1986; Suh, 1987) is not considered in the model.

Let  $F(x,y;\alpha_1,\alpha_2,\beta)$  be the joint distribution function that satisfies the monotone likelihood ratio (MLR) and the convexity of the cumulative distribution function (CDF) conditions. Some Japanese companies have found the natural rhythm of nonpositive correlation between production and quality existing in production systems. It is of no value for the management to press for greater production at the expense of quality levels. Instead, they search for the balance point that will provide the optimum level of production and quality (Lubben, 1988, p. 123). Following the same observation, it is further assumed

that the production and quality levels ( $x$  and  $y$ ) are nonpositively correlated.

The principal can observe both the total output,  $x$ , and its quality,  $y$ . Therefore,  $x$  and  $y$  can be utilized as the basis of a contract. The manager receives a nonnegative payment  $M(x,y)$ , and his expected utility  $G(M(x,y),\beta)$  is defined over that payment and his effort. Following conventional agency assumptions, the agents are risk averse and the disutility of effort increases at a nondecreasing rate (i.e. effort averse).  $G(M(x,y),\beta)$  is assumed to be additively separable, i.e.  $G(M(x,y),\beta) = E[V(M(x,y)) - W_2(\beta)]$ , where  $V'(\cdot) > 0$ ,  $V''(\cdot) < 0$ ,  $W_2'(\cdot) > 0$  and  $W_2''(\cdot) \geq 0$ . The worker also receives a nonnegative payment  $S(x,y)$ , and his expected utility,  $H(S(x,y),\alpha_1,\alpha_2)$ , is defined over his income and the production and quality-enhancing effort.  $H(S(x,y),\alpha_1,\alpha_2)$  is also assumed to be additively separable, i.e.  $H(S(x,y),\alpha_1,\alpha_2) = E[U(S(x,y)) - W_1(k_1\alpha_1 + k_2\alpha_2)]$ , where  $U'(\cdot) > 0$ ,  $U''(\cdot) < 0$ ,  $W_1'(\cdot) > 0$ ,  $W_1''(\cdot) \geq 0$ , and  $k_1, k_2$  are job enlargement factors with  $0 < k_1 \leq 1$  and  $0 < k_2 \leq 1$ . In order to avoid the nonexistence of solutions,  $M(x,y)$  and  $S(x,y)$  are restricted to lie in certain intervals (Holmstrom, 1979).

Given the above assumptions and specifications, the three-level agency model can be set up as the following program:

The principal seeks to

$$(OP) \quad \begin{array}{ll} \text{Max} & E[\pi] = E[p(y)x - c(x) - S(x,y) - M(x,y)] \\ \text{S, M} & \end{array} \quad (3.1)$$

Subject to

$$H[S, \alpha_1, \alpha_2] \equiv E[U(S)] - W_1(k_1\alpha_1 + k_2\alpha_2) \geq H^0 \quad (3.2a)$$

$$\alpha_1, \alpha_2 \in \underset{\alpha_1', \alpha_2'}{\text{Argmax}} \quad \{E[U(S)] - W_1(k_1\alpha_1' + k_2\alpha_2')\} \quad (3.2b)$$

$$G[M, \beta] \equiv E[V(M)] - W_2(\beta) \geq G^\circ \quad (3.3a)$$

$$\beta \in \underset{\beta'}{\text{Argmax}} \{E[V(M)] - W_2(\beta')\} \quad (3.3b)$$

where  $E[\cdot]$  denotes the expectation operator, the notation "argmax" denotes the set of arguments that maximize the indicated objective function, and  $G^\circ$  and  $H^\circ$  are the known market-reservation utility levels of the manager and the worker, respectively.

The principal's expected profit is represented in (3.1). The individual rationality constraints, (3.2a) and (3.3a), guarantee that the worker's and the manager's expected utilities exceed their minimum market levels so that they will be attracted to and stay in the job. The incentive compatibility constraints, (3.2b) and (3.3b), indicate that each agent will select his effort level from his decision set to maximize his own expected utility.

In order to replace constraints (3.2b) and (3.3b) by their corresponding first-order conditions, a supplementary assumption on utility functions must be made. That is,  $U(U'^{-1}(1/S))$  and  $V(V'^{-1}(1/M))$  are concave in  $S$  and  $M$ , where  $U'^{-1}$  and  $V'^{-1}$  denote the inverse function of  $U'$  and  $V'$  (Theorem 2, Jewitt, 1988).

Given that the first-order conditions hold, determination of the contracts  $S(x,y)$  and  $M(x,y)$  can be replaced by the following optimal control program (FP):

$$(FP) \quad \text{Max}_{S,M} \quad \iint [p(y)x - c(x) - S(x,y) - M(x,y)] f(x,y;\alpha_1,\alpha_2,\beta) dx dy \quad (3.4)$$

$$\text{Subject to} \quad \iint U(S(x,y)) f(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_1(k_1\alpha_1 + k_2\alpha_2) \geq H^0 \quad (3.5)$$

$$\iint V(M(x,y)) f(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_2(\beta) \geq G^0 \quad (3.6)$$

$$\iint U(S(x,y)) f_{\alpha_1}(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_1' k_1(k_1\alpha_1 + k_2\alpha_2) = 0 \quad (3.7)$$

$$\iint U(S(x,y)) f_{\alpha_2}(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_1' k_2(k_1\alpha_1 + k_2\alpha_2) = 0 \quad (3.8)$$

$$\iint V(M(x,y)) f_{\beta}(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_2'(\beta) = 0 \quad (3.9)$$

where  $f_i(x,y;\alpha_1,\alpha_2,\beta)$  is the partial derivative of  $f(x,y;\alpha_1,\alpha_2,\beta)$  with respect to  $i$ ,  $i=\alpha_1,\alpha_2,\beta$ ; and  $W_j'(\cdot)$  is the first derivative of  $W_j(\cdot)$ ,  $j=1,2$ .

### Characterization of Optimal Contracts

Let  $\lambda_1$ ,  $\lambda_2$ ,  $\mu_{11}$ ,  $\mu_{12}$ , and  $\mu_2$  be the multipliers associated with constraints (3.5)-(3.9), respectively. The Hamiltonian of the above program (FP) can be written as the following function:

$$\begin{aligned} (H) \quad \text{Max} \quad & L(S,M,\lambda_1,\lambda_2,\mu_{11},\mu_{12},\mu_2,\alpha_1,\alpha_2,\beta) \\ &= \iint [p(y)x - c(x) - S(x,y) - M(x,y)] f(x,y;\alpha_1,\alpha_2,\beta) dx dy \\ &+ \lambda_1 \{ \iint U(S(x,y)) f(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_1(k_1\alpha_1 + k_2\alpha_2) - H^0 \} \\ &+ \mu_{11} \{ \iint U(S(x,y)) f_{\alpha_1}(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_1' k_1(k_1\alpha_1 + k_2\alpha_2) \} \\ &+ \mu_{12} \{ \iint U(S(x,y)) f_{\alpha_2}(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_1' k_2(k_1\alpha_1 + k_2\alpha_2) \} \\ &+ \lambda_2 \{ \iint V(M(x,y)) f(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_2(\beta) - G^0 \} \\ &+ \mu_2 \{ \iint V(M(x,y)) f_{\beta}(x,y;\alpha_1,\alpha_2,\beta) dx dy - W_2'(\beta) \} \end{aligned} \quad (3.10)$$

with  $\lambda_1 \geq 0$  and  $\lambda_2 \geq 0$ .

If the optimal  $S(x,y)$  and  $M(x,y)$  are in the interior of the set of feasible contracts, they can be characterized as the solutions to the

necessary Euler conditions. Thus, the optimality conditions for contracts are

$$\begin{aligned} \frac{1}{U'(S)} &= \lambda_1 + \mu_{11} \frac{f_{\alpha_1}(x, y)}{f(x, y)} + \mu_{12} \frac{f_{\alpha_2}(x, y)}{f(x, y)} \\ &= \lambda_1 + \mu_{11} \frac{f_{\alpha_1}(x|y)}{f(x|y)} + \mu_{12} \frac{f_{\alpha_2}(y|x)}{f(y|x)} \end{aligned} \quad (3.11)$$

$$\begin{aligned} \text{and } \frac{1}{V'(M)} &= \lambda_2 + \mu_2 \frac{f_{\beta}(x, y)}{f(x, y)} \\ &= \lambda_2 + \mu_2 \frac{f_{\beta}(y|x)}{f(y|x)}, \end{aligned} \quad (3.12)$$

where the second equations of (3.11) and (3.12) come from the assumptions that  $x=x(\alpha_1, \theta)$  and  $y=y(\alpha_2, \beta, \epsilon)$ .

Taking the first derivative of (3.10) with respect to  $\alpha_1$ ,  $\alpha_2$ , and  $\beta$ , the following conditions are obtained:

$$\begin{aligned} &\iint [p(y)x - c(x) - S(x, y) - M(x, y)] f_{\alpha_1}(x, y) dx dy \\ &+ \lambda_1 \{ \iint U(S(x, y)) f_{\alpha_1}(x, y) dx dy - k_1 W_1'(k_1 \alpha_1 + k_2 \alpha_2) \} \\ &+ \mu_{11} \{ \iint U(S(x, y)) f_{\alpha_1 \alpha_1}(x, y) dx dy - W_1'' k_1^2 (k_1 \alpha_1 + k_2 \alpha_2) \} \\ &+ \mu_{12} \{ \iint U(S(x, y)) f_{\alpha_1 \alpha_2}(x, y) dx dy - W_1'' k_1 k_2 (k_1 \alpha_1 + k_2 \alpha_2) \} \\ &+ \lambda_2 \{ \iint V(M(x, y)) f_{\alpha_1}(x, y) dx dy \} \\ &+ \mu_2 \{ \iint V(M(x, y)) f_{\beta \alpha_1}(x, y) dx dy \} = 0; \end{aligned} \quad (3.13)$$

$$\begin{aligned}
& \iint [p(y)x - c(x) - S(x, y) - M(x, y)] f_{\alpha_2}(x, y) dx dy \\
& + \lambda_1 \{ \iint U(S(x, y)) f_{\alpha_2}(x, y) dx dy - W_1' k_2(k_1 \alpha_1 + k_2 \alpha_2) \} \\
& + \mu_{11} \{ \iint U(S(x, y)) f_{\alpha_1 \alpha_2}(x, y) dx dy - W_1'' k_1 k_2(k_1 \alpha_1 + k_2 \alpha_2) \} \\
& + \mu_{12} \{ \iint U(S(x, y)) f_{\alpha_2 \alpha_2}(x, y) dx dy - W_1'' k_2^2(k_1 \alpha_1 + k_2 \alpha_2) \} \\
& + \lambda_2 \{ \iint V(M(x, y)) f_{\alpha_2}(x, y) dx dy \} \\
& + \mu_2 \{ \iint V(M(x, y)) f_{\beta \alpha_2}(x, y) dx dy \} = 0; \tag{3.14}
\end{aligned}$$

$$\begin{aligned}
& \iint [p(y)x - c(x) - S(x, y) - M(x, y)] f_{\beta}(x, y) dx dy \\
& + \lambda_1 \{ \iint U(S(x, y)) f_{\beta}(x, y) dx dy \} \\
& + \mu_{11} \{ \iint U(S(x, y)) f_{\alpha_1 \beta}(x, y) dx dy \} \\
& + \mu_{12} \{ \iint U(S(x, y)) f_{\beta \alpha_2}(x, y) dx dy \} \\
& + \lambda_2 \{ \iint V(M(x, y)) f_{\beta}(x, y) dx dy - W_2'(\beta) \} \\
& + \mu_2 \{ \iint V(M(x, y)) f_{\beta \beta}(x, y) dx dy - W_2''(\beta) \} = 0. \tag{3.15}
\end{aligned}$$

The optimal compensation schemes and effort levels can be determined by solving equations (3.5)-(3.9) and (3.11)-(3.15) simultaneously, with  $\lambda_1 \geq 0$  and  $\lambda_2 \geq 0$ . The exact solutions may be very difficult to formulate, but some characteristics of the optimal contracts are analyzed in the following results.

**Result 3.1.** Under the optimal compensation schemes, the expected utilities for the manager and worker are exactly equal to their market-reservation utility levels  $H^\circ$  and  $G^\circ$ , respectively; that is, constraints (3.5) and (3.6) hold as equalities at the optimum, and  $\lambda_1 > 0$ ,  $\lambda_2 > 0$ .

**Proof.** By the Kuhn-Tucker condition:  $\lambda_1 \geq 0$ ,  $\lambda_2 \geq 0$ . If  $\lambda_2 = 0$ , equation (3.12) is rewritten as

$$\frac{1}{V'(M)} = \mu_2 \frac{f_{\beta}(x, y)}{f(x, y)}. \tag{3.12'}$$

The left-hand side of the equation (3.12') is always positive by the risk-averse assumption. Since  $\iint f_\beta(x,y)dx dy = 0$ , this is a contradiction. Hence,  $\lambda_2 > 0$ . Evaluating equation (3.11),  $\lambda_1 > 0$  can be proved similarly. Q.E.D.

Result 3.2. Since the first-best solution cannot be reached, the manager should share some risk for the quality-enhancing process, i.e.  $\mu_2 > 0$ .

Proof. (This proof follows Jewitt (1988).)

Substituting (3.12) into (3.9) gives

$$\iint V(M) [1/V'(M) - \lambda_2] f(x,y) dx dy = \mu_2 W_2'(\beta). \quad (3.3.2a)$$

Using the fact that  $E[f_\beta/f] = \iint (f_\beta/f) f dx dy = \iint f_\beta dx dy = 0$ , (3.12) gives

$$\iint [1/V'(M)] f(x,y) dx dy = \lambda_2. \quad (3.3.2b)$$

From Result 3.1 and equation (3.6),  $\iint V(M(x,y)) f(x,y) dx dy = W_2(\beta) + G^0$ .

Hence, (3.3.2a) states that the covariance of  $V(M)$  and  $1/V'(M)$  is of the sign as  $\mu_2 W_2'(\beta)$ . Since  $V(M)$  and  $1/V'(M)$  are monotone in the same direction, they have a nonnegative covariance. And since  $W_2'(\beta)$  is positive by assumption, it follows that  $\mu_2 \geq 0$ . Furthermore,  $\mu_2 = 0$  can be ruled out, for then  $M(x,y)$  would be constant and this violates equation (3.9). Q.E.D.

Result 3.3. The manager's contract is a strictly increasing function of the quantity and quality produced.

Proof. From (3.12),  $M(x,y)$  can be written as

$$M(x,y) = h(\lambda_2 + \mu_2 (f_\beta/f)), \text{ where } h(\cdot) = (1/V'(\cdot))^{-1}.$$

Since  $V'(\cdot)$  is a strictly decreasing function of  $M(x,y)$ ,  $h(\cdot)$  is strictly increasing with its argument. By the fact that  $\lambda_2$  and  $\mu_2$  are positive as proved in results 3.1 and 3.2, and that  $f_\beta/f$  is an

increasing function of  $x$  and  $y$ ,  $M(x,y)$  is also an increasing function of  $x$  and  $y$ . Q.E.D.

Result 3.4. If  $f_\beta(y|x)/f(y|x)$  is linearly increasing in  $y$  and the conditional expected quality  $E(y|x)$  is a function of  $\beta$ , then

$$f_\beta(y|x)/f(y|x) = [y - E(y|x)] E_\beta(y|x) / \text{Var}(y|x).$$

Proof. Since  $f(x,y) = f(y|x)f(x)$ ,  $f_\beta(x,y) = f_\beta(y|x)f(x) + f(y|x)f_\beta(x) = f_\beta(y|x)f(x)$ . Recall equation (3.12)

$$\frac{1}{V'(M)} = \lambda_2 + \mu_2 \frac{f_\beta(y|x)}{f(y|x)}.$$

Define  $h(y|x) = f_\beta(y|x)/f(y|x)$ . Since  $h(y|x)$  is linearly increasing in  $y$ ,  $h_y(y|x) > 0$  and  $h_{yy}(y|x) = 0$ . It can be shown that  $E[h(y|x)] = \int h(y|x)f(y|x)dy = \int f_\beta(y|x)dy = 0$ . By Jensen's inequality and the assumption that  $h$  is linear,  $E[h(y|x)] = h[E(y|x)] = 0$ . Since  $h_y(y|x) > 0$ ,  $h(y|x) \geq 0$  as  $y \geq E(y|x)$ , and  $h(y|x) < 0$  as  $y \leq E(y|x)$ . This suggests that  $h(y|x) = K(y - E(y|x))$ , where  $E[y|x] = \int yf(y|x)dy$ . Also,  $\int yf_\beta(y|x)dy = E_\beta(y|x)$ . But,  $\int yf_\beta(y|x)dy = \int yh(y|x)f(y|x)dy = K \int y(y - E(y|x))f(y|x)dy = K\{E[y^2|x] - (E(y|x))^2\} = K \text{Var}(y|x)$ . This implies  $K = E_\beta(y|x)/\text{Var}(y|x)$ . Therefore,  $f_\beta(y|x)/f(y|x) = h(y|x) = [y - E(y|x)] E_\beta(y|x)/\text{Var}(y|x)$ .

Q.E.D.

Remark. This result shows that both the conditional expectation and variance are used in the evaluation. The principal may use the revised conditional expected quality  $E(y|x)$  as a standard to give a bonus or impose a penalty. And, the compensation is decreasing with the variance. Thus, the conventional standard-setting and variance



computation is part of the optimal compensation scheme (Baiman and Demski, 1980).

Result 3.5. If there is no correlation between quantity and quality, then the optimal contract for the manager is a nontrivial function of the quality standard alone, i.e.  $M(\cdot)=M(y)$ .

Proof. Since  $f_{\beta}(x,y)/f(x,y)=f_{\beta}(y|x)/f(y|x)=f_{\beta}(y)/f(y)$  when  $x$  and  $y$  are independent, the result follows from (3.12) directly. Q.E.D.

Remark. In this case, the product quantity is noninformative (Holmstrom, 1979; Suh, 1988) regarding the manager's action choice. For the manager's performance evaluation purposes, there is no need to consider the quantity level. That is, the quality standard ( $y$ ) becomes a sufficient statistic for  $(x,y)$  regarding  $\beta$ . If quantity and quality are correlated, even though production is beyond the manager's control, the production quantity tells the principal something about the manager's behavior. Thus, both factors are used in the manager's final evaluation. In addition, the manager tends to act as an advocate for his subordinate (Tirole, 1986). He would like to share some production risk with the worker so that the quality-improving policy may be implemented more effectively and efficiently.

Result 3.6. If the principal can observe the agents' actions or the states of nature directly, then:

- (i) the manager's compensation is a constant times the compensation of his subordinate, the worker;
- (ii) the manager's utility, at the optimum, is a linear transformation function of the worker's; and

(iii) the manager's coefficient of absolute risk aversion is a constant times the worker's.

Proof. When the principal can observe the agents' actions or the states of nature, the first-best solution can be achieved. That is,  $1/U'=\lambda_1$  and  $1/V'=\lambda_2$  with  $\lambda_1>0$  and  $\lambda_2>0$ . Therefore,  $S(x,y)=U'^{-1}(1/\lambda_1)$  and  $M(x,y)=V'^{-1}(1/\lambda_2)$ . The agents are paid by a constant salary, given that the specified effort levels are provided. Otherwise, they receive nothing. Conclusion (i) follows directly.

From (i)  $\lambda_2 V'(M)=\lambda_1 U'(S)$  and  $M=kS$ , where  $k$  is a positive constant, we have that  $V'(kS)=(\lambda_1/\lambda_2)U'(S)$ . (3.3.6a)

Integrating (3.3.6a) over  $S$ ,  $V(\cdot)=aU(\cdot)+b$ , where  $a=k\lambda_1/\lambda_2$ , and  $b$  is a constant of integration. Conclusion (ii) follows directly.

Differentiating both sides of (3.3.6a) with respect to  $S$ , we have  $kV''=(\lambda_1/\lambda_2)U''$ . (3.3.6b)

Define the Arrow-Pratt measure of absolute risk aversion as  $R_v=-V''/V'$  and  $R_u=-U''/U'$ . From (3.3.6a) and (3.3.6b), it follows that  $R_v=(1/k)R_u$ .

Q.E.D.

Remark. Herbert Simon (1957) suggested that "an executive's salary should be  $b$  times the salary of his immediate subordinates, ..."

Simon's suggestion is supported under the availability of the first-best solution. Conclusions (ii) and (iii) are consistent with the relationship of Pareto-optimal risk sharing and the similarity rule of utility functions as given in Ross (1974).

Result 3.7. At the optimum, the worker has to share some risk of  $x$  and  $y$ , i.e.  $\mu_{11} > 0$  and  $\mu_{12} > 0$ .

Proof. This proof is similar to result 3.2. Substituting (3.11) into (3.7) gives

$$\iint U((1/U') - \lambda_1)) f(x, y) dx dy - \mu_{12} \iint U f_{\alpha_2} dx dy = \mu_{11} k_1 W_1'. \quad (3.3.7a)$$

Substituting (3.8) into (3.3.7a) gives

$$\iint U((1/U') - \lambda_1)) f(x, y) dx dy = \mu_{11} k_1 W_1' + \mu_{12} k_2 W_1'. \quad (3.3.7b)$$

By the fact that  $E[f_{\alpha_1}/f] = 0$  and  $E[f_{\alpha_2}/f] = 0$ , (3.11) yields

$$\iint (1/U') f(x, y) dx dy = \lambda_1. \quad (3.3.7c)$$

Equation (3.3.7b) states that the covariance of  $U$  and  $1/U'$  is of the same sign as  $(\mu_{11} k_1 W_1' + \mu_{12} k_2 W_1')$  which should be nonnegative (Jewitt, 1988). Therefore,  $\mu_{11}$  and  $\mu_{12}$  cannot both be negative. We can rule out  $\mu_{11} = 0$ ,  $\mu_{12} < 0$  and  $\mu_{11} < 0$ ,  $\mu_{12} = 0$  cases for the same reason.  $\mu_{11}$  and  $\mu_{12}$  cannot both be zero because of (3.7) and (3.8).

We now consider the following two cases:

1.  $\mu_{11} = 0$ ,  $\mu_{12} > 0$ , or  $\mu_{11} > 0$ ,  $\mu_{12} = 0$ , and
2.  $\mu_{11} > 0$ ,  $\mu_{12} < 0$ , or  $\mu_{11} < 0$ ,  $\mu_{12} > 0$ .

The second terms in (3.13) and in (3.14) are equal to zero by (3.7) and (3.8). The fourth term of (3.13) and the third term of (3.14) are positive by first-order stochastic dominance. The third term of (3.13) and the fourth term of (3.14) are negative by the second-order condition. The remaining terms of (3.13) and (3.14) are positive by the fact that  $M(x, y)$  is increasing in  $x$  and  $y$ , and first-order stochastic dominance. Both cases violate equations (3.13) and (3.14). Hence,  $\mu_{11} > 0$  and  $\mu_{12} > 0$ . Q.E.D.

Result 3.8. The worker's compensation is a strictly increasing function of the quantity and quality of products.

Proof. Proof is similar to that of result 3.6.

Q.E.D.

### The Comparative Statics

In order to obtain greater insight from the model, the following additional assumptions are made:

(a) The production and quality-enhancing processes follow a linear technology, i.e.  $x(\alpha_1, \theta) = \ell(\alpha_1) + \theta$ ,  $y(\alpha_2, \beta, \epsilon) = m(\alpha_2, \beta) + \epsilon$ .  $\theta$  and  $\epsilon$  are bivariate standard normally distributed with covariance  $\sigma_{\theta\epsilon}$ . Since  $\sigma_\theta = \sigma_\epsilon = 1$ ,  $\sigma_{\theta\epsilon} = \rho$  with  $-1 < \rho \leq 0$ . And  $\ell'(\alpha_1) > 0$ ,  $\ell''(\alpha_1) \leq 0$ ,  $m_{\alpha_2} > 0$ ,  $m_\beta > 0$ ,  $m_{\alpha_2\alpha_2} \leq 0$ ,  $m_{\beta\beta} \leq 0$  and  $m_{\beta\alpha_2} \geq 0$ , where the subscripts denote partial derivatives. The joint density function is as follows:

$$f(x, y) = (1/2\pi(1-\rho^2)^{1/2}) \exp\{-1/2(1-\rho^2)[(x-\ell)^2 - 2\rho(x-\ell)(y-m) + (y-m)^2]\}.$$

(b) The agents have power utility functions. Moreover, these utility functions are assumed to be  $V(M) = 2M^{1/2}$  and  $U(S) = 3S^{1/3}$ .

(c) The product price,  $p(y)$ , is a linear function of quality, i.e.  $p(y) = p_0 + py$ , where  $p_0 > 0$  and  $p \geq 0$ . This implies that products with a better quality level will also have higher bargaining power.

(d) The production cost,  $c(x)$ , is a linear function of quantity, i.e.  $c(x) = c_0 + cx$ , where  $c_0 > 0$  and  $c > 0$ .

The normal distribution is totally positive (Jewitt, 1987).

Assumption (b) satisfies the utility shape specified by Jewitt (1988).

The power utility functions are constant relative risk-averse functions with a coefficient of relative risk aversion bounded above one half.

The power utility function also has larger risk tolerance than the exponential, logarithmic and quadratic utility functions. If the principal has a choice, the agent who is increasingly risk averse (in the sense of Arrow-Pratt) should be preferred by the risk-neutral principal, so that the agent is willing to pay a higher risk premium. Assumption (b) implies that the worker is more risk averse than the manager.

From assumptions (a)-(d), the optimality conditions for contracts (3.11) and (3.12) can be rewritten as follows:

$$\begin{aligned}
 1/V' &= \lambda_2 + \mu_2 (m_\beta/(1-\rho^2)) [y-m-\rho(x-\ell)] \\
 &= [\lambda_2 - \mu_2(\rho m_\beta/(1-\rho^2))(m-\ell)] - \mu_2(\rho m_\beta/(1-\rho^2))x + \mu_2(m_\beta/(1-\rho^2))y \quad (3.11') \\
 &= a_2 + b_2x + c_2y;
 \end{aligned}$$

$$\begin{aligned}
 1/U' &= \lambda_1 + \mu_{11}(\ell'/(1-\rho^2)) [x-\ell-\rho(y-m)] + \mu_{12}(m_{\alpha_2}/(1-\rho^2)) [y-m-\rho(x-\ell)] \\
 &= [\lambda_1 - (\mu_{11}\ell'/(1-\rho^2) - \mu_{12}\rho m_{\alpha_2}/(1-\rho^2))\ell + (\mu_{11}\rho\ell'/(1-\rho^2) - \mu_{12}m_{\alpha_2}/(1-\rho^2))m] \\
 &\quad + (\mu_{11}\ell'/(1-\rho^2) - \mu_{12}\rho m_{\alpha_2}/(1-\rho^2))x + (-\mu_{11}\rho\ell'/(1-\rho^2) + \mu_{12}m_{\alpha_2}/(1-\rho^2))y \\
 &= a_1 + b_1x + c_1y, \quad (3.12')
 \end{aligned}$$

$a_i$  where  $a_i$  can be treated as the base salary,  $b_i$  as the commission rate of production, and  $c_i$  as the commission rate of quality ( $i=1$  for the worker,  $i=2$  for the manager). From results 3.1, 3.2, 3.3, 3.7, and 3.8,  $a_i \geq 0$ ,  $b_i > 0$ , and  $c_i > 0$ . The following result can be observed immediately.

Result 3.9. The agents are compensated by a lower salary and higher commission rates when there is a negative correlation between the quantity and quality of products.

Proof. If  $\rho=0$ , the optimality conditions of contracts are

$$1/V' = \lambda_2 + \mu_2 m_\beta (y-m), \text{ and} \quad (3.11'')$$

$$1/U' = \lambda_1 + \mu_{11} \ell' (x-\ell) + \mu_{12} m_{\alpha_2} (y-m). \quad (3.12'')$$

Using the fact that all multipliers are positive and  $M(x,y)$  and  $S(x,y)$  are increasing in their arguments, and comparing (3.11'), (3.12') with (3.11'') and (3.12''), respectively, yields the desired result. Q.E.D.

Remark. When quantity and quality of products are negatively correlated, the principal needs to provide more incentive for the risk-averse agents to overcome this additional uncertainty. A contract with higher commissions and lower salary will induce more effort.

The discussion of comparative statics will utilize the following preliminary results.

#### Preliminaries of The Comparative Statics

For each effort level, the optimal contracts for the manager and the worker are specified by substituting the multipliers into (3.11') and (3.12'). Then, by comparing the solutions across effort levels, the optimal effort levels can be determined.

The manager. From (3.11), (3.11') and the power utility function assumption, we have

$$\begin{aligned} M(x,y) &= [\lambda_2 + \mu_2 f_\beta / f]^2 \\ &= \{\lambda_2 + \mu_2 (m_\beta / (\ell - \rho^2)) [y - m - \rho(x-1)]\}^2. \end{aligned} \quad (3.16)$$

Therefore, the manager's utility function can be written as

$$V(M(x,y)) = 2[\lambda_2 + \mu_2 f_\beta / f]. \quad (3.17)$$

Substituting (3.17) in (3.6) with the equality holding yields

$$\begin{aligned} W_2 + G^\circ &= 2 \iint [\lambda_2 + \mu_2 f_\beta / f] f dx dy \\ &= 2\lambda_2 + 2\mu_2 \iint [f_\beta / f] f dx dy \\ &= 2\lambda_2. \end{aligned}$$

$$\text{Therefore, } \lambda_2 = (W_2 + G^\circ) / 2. \quad (3.18)$$

From (3.9) and (3.17), we have

$$\begin{aligned}
 W_2' &= 2 \iint [\lambda_2 + \mu_2 f_\beta / f] f_\beta dx dy \\
 &= 2\mu_2 \iint [f_\beta / f]^2 f dx dy \\
 &= 2\mu_2 \iint \{ (m_\beta / (1 - \rho^2)) [y - m - \rho(x - \ell)] \}^2 f dx dy \\
 &= 2\mu_2 (m_\beta / (1 - \rho^2))^2 \{ E(y - m)^2 - 2\rho E[(y - m)(x - \ell)] + \rho^2 E(x - \ell)^2 \} \\
 &= 2\mu_2 (m_\beta)^2 / (1 - \rho^2),
 \end{aligned}$$

$$\text{or } \mu_2 = W_2' ((1 - \rho^2) / 2(m_\beta)^2). \quad (3.19)$$

Substituting  $\lambda_2$  and  $\mu_2$  into (3.16), we have

$$\begin{aligned}
 M(x, y) &= \{ [(W_2 + G^\circ) / 2 - (W_2' / 2m_\beta)(m - \rho\ell)] - (\rho W_2' / 2m_\beta)x + (W_2' / 2m_\beta)y \}^2 \\
 &= (A_2 + B_2x + C_2y)^2,
 \end{aligned} \quad (3.20)$$

where  $A_2 = (W_2 + G^\circ) / 2 - (W_2' / 2m_\beta)(m - \rho\ell)$ ,  $B_2 = -\rho W_2' / 2m_\beta$  and  $C_2 = W_2' / 2m_\beta$ .

Therefore, the expected compensation for the manager is

$$E[M(x, y)] = (1 / 2m_\beta)^2 \iint \{ A_2 + B_2x + C_2y \}^2 f dx dy = ((W_2 + G^\circ) / 2)^2 + (W_2' / 2m_\beta)^2 (1 - \rho^2). \quad (3.21)$$

The worker. From (3.12), (3.12') and the power utility function assumption, we have

$$\begin{aligned}
 S(x, y) &= [\lambda_1 + \mu_{11} f_{\alpha_1} / f + \mu_{12} f_{\alpha_2} / f]^3 \\
 &= \{ \lambda_1 + \mu_{11} (\ell' / (1 - \rho^2)) [x - \ell - \rho(y - m)] + \mu_{12} (m_{\alpha_2} / (1 - \rho^2)) [y - m - \rho(x - \ell)] \}^3.
 \end{aligned} \quad (3.22)$$

Therefore, the manager's utility function can be written as

$$U(S(x, y)) = 3[\lambda_1 + \mu_{11} f_{\alpha_1} / f + \mu_{12} f_{\alpha_2} / f]. \quad (3.23)$$

Substituting (3.23) into (3.5) with the equality holding yields

$$\begin{aligned}
 W_1 + H^\circ &= 3 \iint [\lambda_1 + \mu_{11} f_{\alpha_1} / f + \mu_{12} f_{\alpha_2} / f] f dx dy \\
 &= 3\lambda_1.
 \end{aligned}$$

$$\text{Therefore, } \lambda_1 = (W_1 + H^\circ) / 3. \quad (3.24)$$

From (3.7) and (3.23), we have

$$\begin{aligned} k_1 W_1' &= 3 \iint [\lambda_1 + \mu_{11} f_{\alpha_1} / f + \mu_{12} f_{\alpha_2} / f_{\alpha_1}] dx dy \\ &= 3 \mu_{11} (\ell')^2 / (1 - \rho^2), \end{aligned}$$

$$\text{or} \quad \mu_{11} = k_1 W_1' ((1 - \rho^2) / 3 (\ell')^2). \quad (3.25)$$

From (3.8) and (3.23), similarly we have

$$\mu_{12} = k_2 W_1' ((1 - \rho^2) / 3 (m_{\alpha_2})^2). \quad (3.26)$$

Substituting  $\lambda_1$ ,  $\mu_{11}$ , and  $\mu_{12}$  into (3.22), we obtain

$$\begin{aligned} S(x, y) &= \{ (W_1 + H^0) / 3 + k_1 W_1' (1 - \rho^2) / 3 (\ell')^2 (\ell' / (1 - \rho^2)) [x - \ell - \rho(y - m)] \\ &\quad + k_2 W_1' (1 - \rho^2) / 3 (m_{\alpha_2})^2 (m_{\alpha_2} / (1 - \rho^2)) [y - m - \rho(x - \ell)] \}^3 \\ &= \{ (W_1 + H^0) / 3 + [k_1 W_1' / 3 \ell' - \rho k_2 W_1' / 3 m_{\alpha_2}] (x - \ell) \\ &\quad + [-\rho k_1 W_1' / 3 \ell' + k_2 W_1' / 3 m_{\alpha_2}] (y - m) \}^3 \\ &= \{ A_1 + B_1 x + C_1 y \}^3 \end{aligned} \quad (3.27)$$

where  $A_1 = (W_1 + H^0) / 3 - (k_1 W_1' / 3 \ell') (\ell - \rho m) - (k_2 W_1' / 3 m_{\alpha_2}) (m - \rho \ell)$

$$B_1 = k_1 W_1' / 3 \ell' - \rho k_2 W_1' / 3 m_{\alpha_2}$$

$$C_1 = -\rho k_1 W_1' / 3 \ell' + k_2 W_1' / 3 m_{\alpha_2}.$$

Therefore, the worker's expected compensation is

$$\begin{aligned} E[S(x, y)] &= ((W_1 + H^0) / 3)^3 \\ &\quad + (W_1 + H^0) (W_1' / 3 \ell' m_{\alpha_2})^2 (1 - \rho^2) [(k_1 m_{\alpha_2})^2 + (k_2 \ell')^2 - 2 \rho k_1 k_2 \ell' m_{\alpha_2}]. \end{aligned} \quad (3.28)$$

The principal. The principal's expected revenue and profit can be written as

$$E[p(y)x - c(x)] = E[(p_0 + py)x - (c_0 + cx)] = [(p_0 + pm - c)\ell - c_0 + \rho p] \quad (3.29)$$

$$\begin{aligned} E[\pi | \alpha_1, \alpha_2, \beta] &= E[(p_0 + py)x - (c_0 + cx)] - E[M(x, y)] - E[S(x, y)] \\ &= [(p_0 + pm - c)\ell - c_0 + \rho p] - ((W_2 + G^0) / 2)^2 - (W_2' / 2 m_\beta)^2 (1 - \rho^2) - (W_1 + H^0 / 3)^3 - \\ &\quad (W_1 + H^0) (W_1' / 3 \ell' m_{\alpha_2})^2 (1 - \rho^2) [(k_1 m_{\alpha_2})^2 + (k_2 \ell')^2 - 2 \rho k_1 k_2 \ell' m_{\alpha_2}]. \end{aligned} \quad (3.30)$$

The effort level. Since  $W_i$  and  $(W_i)^2$  are convex by assumption, the principal's expected profit,  $E[\pi | \alpha_1, \alpha_2, \beta]$ , is a concave function of



$\alpha_1$ ,  $\alpha_2$ , and  $\beta$ . Hence, the unique  $\alpha_1^*$ ,  $\alpha_2^*$ , and  $\beta^*$  that maximizes the  $E[\pi]$  exist.

From (3.30), the optimal effort levels,  $\alpha_1$ ,  $\alpha_2$ ,  $\beta$  are such that

$$\begin{aligned}
 T_1(\alpha_1) &= E_{\alpha_1}[\pi | \alpha_1, \alpha_2, \beta] \\
 &= (pm-c)\ell' - ((W_1+H^\circ)/3)^2(k_1W_1') \\
 &\quad - \{ [k_1W_1'(1-\rho^2)/(3\ell'm_{\alpha_2})^2] [(W_1')^2 + 2(W_1+H^\circ)W_1''] \\
 &\quad \quad [ (k_1m_{\alpha_2})^2 + (k_2\ell')^2 - 2\rho k_1k_2\ell'm_{\alpha_2}] \} \\
 &= 0.
 \end{aligned} \tag{3.31}$$

$$T_{11}(\alpha_1) = E_{\alpha_1\alpha_1}[\pi | \alpha_1, \alpha_2, \beta] < 0.$$

$$\begin{aligned}
 T_2(\alpha_2) &= E_{\alpha_2}[\pi | \alpha_1, \alpha_2, \beta] \\
 &= (p\ell m_{\alpha_2}) - ((W_1+H^\circ)/3)^2(k_2W_1') \\
 &\quad - \{ [k_2W_1'(1-\rho^2)/(3\ell'm_{\alpha_2})^2] [(W_1')^2 + 2(W_1+H^\circ)W_1''] \\
 &\quad \quad [ (k_1m_{\alpha_2})^2 + (k_2\ell')^2 - 2\rho k_1k_2\ell'm_{\alpha_2}] \} \\
 &= 0.
 \end{aligned} \tag{3.32}$$

$$T_{22}(\alpha_2) = E_{\alpha_2\alpha_2}[\pi | \alpha_1, \alpha_2, \beta] < 0.$$

$$\begin{aligned}
 T_3(\beta) &= E_\beta[\pi | \alpha_1, \alpha_2, \beta] \\
 &= \rho\ell m_\beta - (W_2'(W_2+G^\circ)/2)) - (1-\rho^2)(W_2'W_2''/2(m_\beta)^2) \\
 &\quad + 2m_{\beta\alpha_2}(1-\rho^2)(W_1+H^\circ)(W_1'/3\ell')^2(1/m_{\alpha_2})^3 [ (k_1m_{\alpha_2})^2 + (k_2\ell')^2 - 2\rho k_1k_2\ell'm_{\alpha_2}] \\
 &\quad - 2(1-\rho^2)(W_1+H^\circ)(W_1'/3\ell'm_{\alpha_2})^2 [k_1m_{\alpha_2} - \rho k_2\ell'] k_1m_{\beta\alpha_2} \\
 &= 0.
 \end{aligned} \tag{3.33}$$

$$T_{33}(\beta) = E_{\beta\beta}[\pi | \alpha_1, \alpha_2, \beta] < 0.$$

In order to facilitate the computation, it is assumed that

$\ell(\alpha_1) = \alpha_1$ , and  $m(\alpha_2, \beta) = \alpha_2 + \beta$ . For the manager, from (3.20) and (3.21), we have

$$\partial A_2 / \partial \alpha_1 = \rho W_2' / 2 < 0.$$

$$\partial A_2 / \partial \alpha_2 = -W_2' / 2 < 0.$$

$$\partial A_2/\partial \beta = -(W_2''(\beta + \alpha_2 - \rho\alpha_1))/2 < 0.$$

$$\partial B_2/\partial \alpha_1 = 0, \quad \partial B_2/\partial \alpha_2 = 0, \quad \partial B_2/\partial \beta = -\rho W_2''/2 > 0.$$

$$\partial C_2/\partial \alpha_1 = 0, \quad \partial C_2/\partial \alpha_2 = 0, \quad \partial C_2/\partial \beta = W_2''/2 > 0.$$

$$\partial E[M]/\partial \alpha_1 = 0, \quad \partial E[M]/\partial \alpha_2 = 0,$$

$$\partial E[M]/\partial \beta = (W_2'(W_2 + G^\circ)/2) + (W_2'W_2''(1-\rho^2))/2 > 0.$$

For the worker, from (3.27) and (3.28), we have

$$\partial A_1/\partial \alpha_1 = -k_1^2 W_1''(\alpha_1 - \rho\alpha_2 - \rho\beta)/3 - k_1 k_2 W_1''(\alpha_2 - \rho\alpha_1 + \beta)/3 + \rho k_2 W_1'/3 < 0.$$

$$\partial A_1/\partial \alpha_2 = -k_1 k_2 W_1''(\alpha_1 - \rho\alpha_2 - \rho\beta)/3 - k_1 k_2 W_1''(\alpha_2 - \rho\alpha_1 + \beta)/3 + \rho k_1 W_1'/3 < 0.$$

$$\partial A_1/\partial \beta = \rho k_1 W_1'/3 - k_2 W_1'/3 < 0.$$

$$\partial B_1/\partial \alpha_1 = (k_1^2 - \rho k_1 k_2) W_1''/3 > 0.$$

$$\partial B_1/\partial \alpha_2 = (-\rho k_2^2 + k_1 k_2) W_1''/3 > 0.$$

$$\partial B_1/\partial \beta = 0.$$

$$\partial C_1/\partial \alpha_1 = (-\rho k_1^2 + k_1 k_2) W_1''/3 > 0.$$

$$\partial C_1/\partial \alpha_2 = (k_2^2 - \rho k_1 k_2) W_1''/3 > 0.$$

$$\partial C_1/\partial \beta = 0.$$

$$\begin{aligned} \partial E[S]/\partial \alpha_1 = & ((W_1 + H^\circ)/3)^2 k_1 W_1' + (1-\rho^2)(k_1^2 + k_2^2 - 2\rho k_1 k_2)(1/3)^2 \{k_1 (W_1')^2 (W_1 + H^\circ) \\ & + 2k_1 W_1' W_1'' (W_1 + H^\circ)\} > 0. \end{aligned}$$

$$\begin{aligned} \partial E[S]/\partial \alpha_2 = & ((W_1 + H^\circ)/3)^2 k_2 W_1' + (1-\rho^2)(k_1^2 + k_2^2 - 2\rho k_1 k_2)(1/3)^2 \{k_2 (W_1')^2 (W_1 + H^\circ) \\ & + 2k_2 W_1' W_1'' (W_1 + H^\circ)\} > 0. \end{aligned}$$

$$\partial E[S]/\partial \beta = 0.$$

The effect of changing the quality mark-up price ( $p$ ), the marginal production cost ( $c$ ), the manager's market reservation level ( $G^\circ$ ), the worker's market reservation level ( $H^\circ$ ), and the job-enlargement factor ( $k_1$  or  $k_2$ ) are discussed as follows.

### The Effect of The Quality Mark-up Price (p)

Result 3.10. Both agents' efforts increase with an increase of the quality mark-up price. Agents will be paid by contracts with lower salary and higher commission rates. The principal's expected profit and the agents' expected compensation are also increased.

Proof. Holding all other parameters fixed, only the effects of a change in the quality mark-up price parameter (p) are considered.

$$d\alpha_1/dp = -(\partial T_1/\partial p)/(\partial T_1/\partial \alpha_1), \text{ sign}[d\alpha_1/dp] = \text{sign}[\partial T_1/\partial p] = \text{sign}[\alpha_2 + \beta] > 0.$$

$$d\alpha_2/dp = -(\partial T_2/\partial p)/(\partial T_2/\partial \alpha_1), \text{ sign}[d\alpha_2/dp] = \text{sign}[\partial T_2/\partial p] = \text{sign}[\alpha_1] > 0.$$

$$d\beta/dp = -(\partial T_3/\partial p)/(\partial T_3/\partial \beta), \text{ sign}[d\beta/dp] = \text{sign}[\partial T_3/\partial p] = \text{sign}[\alpha_1] > 0.$$

$$dA_1/dp = (\partial A_1/\partial p) + (\partial A_1/\partial \alpha_1)(d\alpha_1/dp) + (\partial A_1/\partial \alpha_2)(d\alpha_2/dp) + (\partial A_1/\partial \beta)(d\beta/dp) < 0.$$

$$dB_1/dp = (\partial B_1/\partial p) + (\partial B_1/\partial \alpha_1)(d\alpha_1/dp) + (\partial B_1/\partial \alpha_2)(d\alpha_2/dp) + (\partial B_1/\partial \beta)(d\beta/dp) > 0.$$

$$dC_1/dp = (\partial C_1/\partial p) + (\partial C_1/\partial \alpha_1)(d\alpha_1/dp) + (\partial C_1/\partial \alpha_2)(d\alpha_2/dp) + (\partial C_1/\partial \beta)(d\beta/dp) > 0.$$

$$\begin{aligned} dE(S)/dp &= (\partial E(S)/\partial p) + (\partial E(S)/\partial \alpha_1)(d\alpha_1/dp) + (\partial E(S)/\partial \alpha_2)(d\alpha_2/dp) \\ &\quad + (\partial E(S)/\partial \beta)(d\beta/dp) > 0. \end{aligned}$$

$$dA_2/dp = (\partial A_2/\partial p) + (\partial A_2/\partial \alpha_1)(d\alpha_1/dp) + (\partial A_2/\partial \alpha_2)(d\alpha_2/dp) + (\partial A_2/\partial \beta)(d\beta/dp) < 0.$$

$$dB_2/dp = (\partial B_2/\partial p) + (\partial B_2/\partial \alpha_1)(d\alpha_1/dp) + (\partial B_2/\partial \alpha_2)(d\alpha_2/dp) + (\partial B_2/\partial \beta)(d\beta/dp) > 0.$$

$$dC_2/dp = (\partial C_2/\partial p) + (\partial C_2/\partial \alpha_1)(d\alpha_1/dp) + (\partial C_2/\partial \alpha_2)(d\alpha_2/dp) + (\partial C_2/\partial \beta)(d\beta/dp) > 0.$$

$$\begin{aligned} dE(M)/dp &= (\partial E(M)/\partial p) + (\partial E(M)/\partial \alpha_1)(d\alpha_1/dp) + (\partial E(M)/\partial \alpha_2)(d\alpha_2/dp) \\ &\quad + (\partial E(M)/\partial \beta)(d\beta/dp) > 0. \end{aligned}$$

$$dE(\pi)/dp = \partial E(\pi)/\partial p = \alpha_1(\alpha_2 + \beta) > 0. \quad \text{Q.E.D.}$$

Remark. The risk-neutral principal's marginal revenue increases with an increase of the quality mark-up price. Therefore, he would like to induce higher effort levels. For the risk-averse agents, the increase of commission rates provides stronger incentive than the fixed salary to induce efforts. Although the expected compensation for agents is

increased, the principal receives even a higher revenue. Therefore, the expected profit is also increased.

### The Effect of The Marginal Production Cost (c)

**Result 3.11.** The worker's production effort decreases with an increase in the marginal production cost. Both agents' salaries are increased, but the worker's commission rates are decreased. The principal's expected profit and the worker's expected compensation are both decreased, but the manager's expected payment remains unchanged. Proof. Holding all other parameters fixed, only the effects of a change in the marginal production cost (c) are considered.

$$d\alpha_1/dc = -(\partial T_1/\partial c)/(\partial T_1/\partial \alpha_1), \text{ sign}[d\alpha_1/dc] = \text{sign}[\partial T_1/\partial c] = \text{sign}[-1] < 0.$$

$$d\alpha_2/dc = -(\partial T_2/\partial c)/(\partial T_2/\partial \alpha_1), \text{ sign}[d\alpha_2/dc] = \text{sign}[\partial T_2/\partial c] = 0.$$

$$d\beta/dc = -(\partial T_3/\partial c)/(\partial T_3/\partial \beta), \text{ sign}[d\beta/dc] = \text{sign}[\partial T_3/\partial c] = 0.$$

$$dA_1/dc = (\partial A_1/\partial c) + (\partial A_1/\partial \alpha_1)(d\alpha_1/dc) + (\partial A_1/\partial \alpha_2)(d\alpha_2/dc) + (\partial A_1/\partial \beta)(d\beta/dc) > 0.$$

$$dB_1/dc = (\partial B_1/\partial c) + (\partial B_1/\partial \alpha_1)(d\alpha_1/dc) + (\partial B_1/\partial \alpha_2)(d\alpha_2/dc) + (\partial B_1/\partial \beta)(d\beta/dc) < 0.$$

$$dC_1/dc = (\partial C_1/\partial c) + (\partial C_1/\partial \alpha_1)(d\alpha_1/dc) + (\partial C_1/\partial \alpha_2)(d\alpha_2/dc) + (\partial C_1/\partial \beta)(d\beta/dc) < 0.$$

$$dE(S)/dc = (\partial E(S)/\partial c) + (\partial E(S)/\partial \alpha_1)(d\alpha_1/dc) + (\partial E(S)/\partial \alpha_2)(d\alpha_2/dc) \\ + (\partial E(S)/\partial \beta)(d\beta/dc) < 0.$$

$$dA_2/dc = (\partial A_2/\partial c) + (\partial A_2/\partial \alpha_1)(d\alpha_1/dc) + (\partial A_2/\partial \alpha_2)(d\alpha_2/dc) + (\partial A_2/\partial \beta)(d\beta/dc) > 0.$$

$$dB_2/dc = (\partial B_2/\partial c) + (\partial B_2/\partial \alpha_1)(d\alpha_1/dc) + (\partial B_2/\partial \alpha_2)(d\alpha_2/dc) + (\partial B_2/\partial \beta)(d\beta/dc) = 0.$$

$$dC_2/dc = (\partial C_2/\partial c) + (\partial C_2/\partial \alpha_1)(d\alpha_1/dc) + (\partial C_2/\partial \alpha_2)(d\alpha_2/dc) + (\partial C_2/\partial \beta)(d\beta/dc) = 0.$$

$$dE(M)/dc = (\partial E(M)/\partial c) + (\partial E(M)/\partial \alpha_1)(d\alpha_1/dc) + (\partial E(M)/\partial \alpha_2)(d\alpha_2/dc) \\ + (\partial E(M)/\partial \beta)(d\beta/dc) = 0.$$

$$dE(\pi)/dc = \partial E(\pi)/\partial c = -\alpha_1 < 0.$$

Q.E.D.

Remark. While the marginal production cost is increasing, the principal's expected marginal revenue is decreasing. On the one hand, the principal would like to induce lower production effort. A contract with lower commissions reduces the worker's desire to produce. On the other hand, the principal does not want to reduce the quality standard. Thus, a higher salary will offset the negative effect so that the manager's effort and expected compensation remain unchanged.

### The Effect of The Manager's Market Reservation Utility ( $G^\circ$ )

Result 3.12. The manager's effort and commission rates decrease as his market reservation level increases. Both agents' salaries are increased. The principal's expected profit is decreased with this change. The manager's expected compensation is decreased provided that his marginal disutility is greater than  $\sqrt{2}$ . The worker's expected compensation remains unchanged.

Proof. Holding all other parameters fixed, only the effects of a change in the manager's market value are considered.

$$d\alpha_1/dG^\circ = -(\partial T_1/\partial G^\circ)/(\partial T_1/\partial \alpha_1), \text{ sign}[d\alpha_1/dG^\circ] = \text{sign}[\partial T_1/\partial G^\circ] = 0.$$

$$d\alpha_2/dG^\circ = -(\partial T_2/\partial G^\circ)/(\partial T_2/\partial \alpha_1), \text{ sign}[d\alpha_2/dG^\circ] = \text{sign}[\partial T_2/\partial G^\circ] = 0.$$

$$d\beta/dG^\circ = -(\partial T_3/\partial G^\circ)/(\partial T_3/\partial \beta), \text{ sign}[d\beta/dG^\circ] = \text{sign}[\partial T_3/\partial G^\circ] = \text{sign}[-W_2'/2] < 0.$$

$$\begin{aligned} dA_1/dG^\circ &= (\partial A_1/\partial G^\circ) + (\partial A_1/\partial \alpha_1)(d\alpha_1/dG^\circ) + (\partial A_1/\partial \alpha_2)(d\alpha_2/dG^\circ) + (\partial A_1/\partial \beta)(d\beta/dG^\circ) \\ &> 0. \end{aligned}$$

$$\begin{aligned} dB_1/dG^\circ &= (\partial B_1/\partial G^\circ) + (\partial B_1/\partial \alpha_1)(d\alpha_1/dG^\circ) + (\partial B_1/\partial \alpha_2)(d\alpha_2/dG^\circ) + (\partial B_1/\partial \beta)(d\beta/dG^\circ) \\ &= 0. \end{aligned}$$

$$\begin{aligned} dC_1/dG^\circ &= (\partial C_1/\partial G^\circ) + (\partial C_1/\partial \alpha_1)(d\alpha_1/dG^\circ) + (\partial C_1/\partial \alpha_2)(d\alpha_2/dG^\circ) + (\partial C_1/\partial \beta)(d\beta/dG^\circ) \\ &= 0. \end{aligned}$$

$$dE(S)/dG^{\circ} = (\partial E(S)/\partial G^{\circ}) + (\partial E(S)/\partial \alpha_1)(d\alpha_1/dG^{\circ}) + (\partial E(S)/\partial \alpha_2)(d\alpha_2/dG^{\circ}) \\ + (\partial E(S)/\partial \beta)(d\beta/dG^{\circ}) = 0.$$

$$dA_2/dG^{\circ} = (\partial A_2/\partial G^{\circ}) + (\partial A_2/\partial \alpha_1)(d\alpha_1/dG^{\circ}) + (\partial A_2/\partial \alpha_2)(d\alpha_2/dG^{\circ}) + (\partial A_2/\partial \beta)(d\beta/dG^{\circ}) \\ > 0.$$

$$dB_2/dG^{\circ} = (\partial B_2/\partial G^{\circ}) + (\partial B_2/\partial \alpha_1)(d\alpha_1/dG^{\circ}) + (\partial B_2/\partial \alpha_2)(d\alpha_2/dG^{\circ}) + (\partial B_2/\partial \beta)(d\beta/dG^{\circ}) \\ < 0.$$

$$dC_2/dG^{\circ} = (\partial C_2/\partial G^{\circ}) + (\partial C_2/\partial \alpha_1)(d\alpha_1/dG^{\circ}) + (\partial C_2/\partial \alpha_2)(d\alpha_2/dG^{\circ}) + (\partial C_2/\partial \beta)(d\beta/dG^{\circ}) \\ < 0.$$

$$dE(M)/dG^{\circ} = (\partial E(M)/\partial G^{\circ}) + (\partial E(M)/\partial \alpha_1)(d\alpha_1/dG^{\circ}) + (\partial E(M)/\partial \alpha_2)(d\alpha_2/dG^{\circ}) \\ + (\partial E(M)/\partial \beta)(d\beta/dG^{\circ}) < 0 \text{ (if } W_2' > \sqrt{2}).$$

$$dE(\pi)/dG^{\circ} = \partial E(\pi)/\partial G^{\circ} = -(W_2 + G^{\circ})/2 < 0. \quad \text{Q.E.D.}$$

Remark. When the manager's market value increases, it is more costly to induce a given level of effort. The principal's marginal revenue remains unchanged. Thus, the principal would like to induce less effort from the manager by reducing the commission rates. In the meantime, the principal does not want to reduce production. A higher salary keeps the worker's efforts at the same levels. Thus, the worker's expected compensation remains unchanged.

### The Effect of The Worker's Market Reservation Utility ( $H^{\circ}$ )

**Result 3.13.** The worker's production and quality-enhancing efforts along with the commission rate decrease with an increase in his market reservation level. Both agents' salaries are increased. The principal's expected profit is decreased and the manager's expected compensation remains unchanged.

Proof. Holding all other parameters fixed, only the effects of a change in the worker's market value are considered.

$$\begin{aligned} d\alpha_1/dH^\circ &= -(\partial T_1/\partial H^\circ)/(\partial T_1/\partial \alpha_1), \text{ sign}[d\alpha_1/dH^\circ] = \text{sign}[T_1/\partial H^\circ] \\ &= \text{sign}[(-2/3)((W_1+H^\circ)/3)(k_1W_1') - (2k_1W_1'W_1''(1-\rho^2)(k_1^2+k_2^2-2\rho k_1k_2)/9)] < 0. \end{aligned}$$

$$\begin{aligned} d\alpha_2/dH^\circ &= -(\partial T_2/\partial H^\circ)/(\partial T_2/\partial \alpha_1), \text{ sign}[d\alpha_2/dH^\circ] = \text{sign}[\partial T_2/\partial H^\circ] \\ &= \text{sign}[(-2/3)((W_1+H^\circ)/3)(k_2W_1') - (2k_2W_1'W_1''(1-\rho^2)(k_1^2+k_2^2-2\rho k_1k_2)/9)] < 0. \end{aligned}$$

$$d\beta/dH^\circ = -(\partial T_3/\partial H^\circ)/(\partial T_3/\partial \beta), \text{ sign}[d\beta/dH^\circ] = \text{sign}[\partial T_3/\partial H^\circ] = 0.$$

$$\begin{aligned} dA_1/dH^\circ &= (\partial A_1/\partial H^\circ) + (\partial A_1/\partial \alpha_1)(d\alpha_1/dH^\circ) + (\partial A_1/\partial \alpha_2)(d\alpha_2/dH^\circ) + (\partial A_1/\partial \beta)(d\beta/dH^\circ) \\ &> 0. \end{aligned}$$

$$\begin{aligned} dB_1/dH^\circ &= (\partial B_1/\partial H^\circ) + (\partial B_1/\partial \alpha_1)(d\alpha_1/dH^\circ) + (\partial B_1/\partial \alpha_2)(d\alpha_2/dH^\circ) + (\partial B_1/\partial \beta)(d\beta/dH^\circ) \\ &< 0. \end{aligned}$$

$$\begin{aligned} dC_1/dH^\circ &= (\partial C_1/\partial H^\circ) + (\partial C_1/\partial \alpha_1)(d\alpha_1/dH^\circ) + (\partial C_1/\partial \alpha_2)(d\alpha_2/dH^\circ) + (\partial C_1/\partial \beta)(d\beta/dH^\circ) \\ &< 0. \end{aligned}$$

$$\begin{aligned} dE(S)/dH^\circ &= (\partial E(S)/\partial H^\circ) + (\partial E(S)/\partial \alpha_1)(d\alpha_1/dH^\circ) + (\partial E(S)/\partial \alpha_2)(d\alpha_2/dH^\circ) \\ &\quad + (\partial E(S)/\partial \beta)(d\beta/dH^\circ) = ? \end{aligned}$$

$$\begin{aligned} dA_2/dH^\circ &= (\partial A_2/\partial H^\circ) + (\partial A_2/\partial \alpha_1)(d\alpha_1/dH^\circ) + (\partial A_2/\partial \alpha_2)(d\alpha_2/dH^\circ) + (\partial A_2/\partial \beta)(d\beta/dH^\circ) \\ &> 0. \end{aligned}$$

$$\begin{aligned} dB_2/dH^\circ &= (\partial B_2/\partial H^\circ) + (\partial B_2/\partial \alpha_1)(d\alpha_1/dH^\circ) + (\partial B_2/\partial \alpha_2)(d\alpha_2/dH^\circ) + (\partial B_2/\partial \beta)(d\beta/dH^\circ) \\ &= 0. \end{aligned}$$

$$\begin{aligned} dC_2/dH^\circ &= (\partial C_2/\partial H^\circ) + (\partial C_2/\partial \alpha_1)(d\alpha_1/dH^\circ) + (\partial C_2/\partial \alpha_2)(d\alpha_2/dH^\circ) + (\partial C_2/\partial \beta)(d\beta/dH^\circ) \\ &= 0. \end{aligned}$$

$$\begin{aligned} dE(M)/dH^\circ &= (\partial E(M)/\partial H^\circ) + (\partial E(M)/\partial \alpha_1)(d\alpha_1/dH^\circ) + (\partial E(M)/\partial \alpha_2)(d\alpha_2/dH^\circ) \\ &\quad + (\partial E(M)/\partial \beta)(d\beta/dH^\circ) = 0. \end{aligned}$$

$$dE(\pi)/dH^\circ = \partial E(\pi)/\partial H^\circ = -((W_2+H^\circ)/3)^2 - (1-\rho^2)(W_1'/3)^2(k_1^2+k_2^2-2\rho k_1k_2) < 0.$$

Q.E.D.

Remark. The intuition behind this result is similar to that behind result 3.12. The worker's compensation cannot be determined, due to the conflicting effects of salary and commissions.

#### The Effect of Job-Enlargement Factor ( $k_1$ or $k_2$ )

Result 3.14. The worker's effort ( $\alpha_1, \alpha_2$ ) devoted to the production and quality-enhancing processes decreases with a decrease in the job-enlargement factor (i.e. an increase in  $k_1$  or  $k_2$ ). The manager's fixed salary is increased, but, his expected compensation and effort level remain unchanged. And, the principal's expected profit is decreased.

Proof. Holding all other parameters fixed, only the effects of a change in the job-enlargement factor  $k_1$  are considered.

$$\begin{aligned} d\alpha_1/dk_1 &= -(\partial T_1/\partial k_1)/(\partial T_1/\partial \alpha_1), \text{ sign}[d\alpha_1/dk_1] = \text{sign}[\partial T_1/\partial k_1] \\ &= \text{sign}\{-2((W_1+H^0)\alpha_1/9)k_1(W_1')^2 - ((W_1+H^0)/3)^2(W_1'+k_1\alpha_1W_1'') \\ &\quad - ((1-\rho^2)(W_1'+k_1\alpha_1W_1'')/9)[(W_1')^2+2W_1''(W_1+H^0)](k_1^2+k_2^2-2\rho k_1k_2) \\ &\quad - (k_1W_1'(1-\rho^2)/9)[2\alpha_1W_1'W_1''+2\alpha_1(W_1'')^2](k_1^2+k_2^2-2\rho k_1k_2) \\ &\quad - (k_1W_1'(1-\rho^2)/9)[(W_1')^2+2W_1''(W_1+H^0)][2k_1-2\rho k_2]\} < 0. \end{aligned}$$

$$\begin{aligned} d\alpha_2/dk_1 &= -(\partial T_2/\partial k_1)/(\partial T_2/\partial \alpha_1), \text{ sign}[d\alpha_2/dk_1] = \text{sign}[\partial T_2/\partial k_1] \\ &= \text{sign}\{-2((W_1+H^0)\alpha_1)/9)k_2(W_1')^2 - ((W_1+H^0)/3)^2(k_2\alpha_1W_1'') \\ &\quad - ((1-\rho^2)k_2\alpha_1W_1''/9)[(W_1')^2+2W_1''(W_1+H^0)](k_1^2+k_2^2-2\rho k_1k_2) \\ &\quad - (k_2W_1'(1-\rho^2)/9)[2\alpha_1W_1'W_1''+2\alpha_1(W_1'')^2](k_1^2+k_2^2-2\rho k_1k_2) \\ &\quad - (k_2W_1'(1-\rho^2)/9)[(W_1')^2+2W_1''(W_1+H^0)][2k_1-2\rho k_2]\} < 0. \end{aligned}$$

$$d\beta/dk_1 = -(\partial T_3/\partial k_1)/(\partial T_3/\partial \beta), \text{ sign}[d\beta/dk_1] = \text{sign}[\partial T_3/\partial k_1] = 0.$$



$$\begin{aligned} dA_1/dk_1 &= (\partial A_1/\partial k_1) + (\partial A_1/\partial \alpha_1)(d\alpha_1/dk_1) + (\partial A_1/\partial \alpha_2)(d\alpha_2/dk_1) + (\partial A_1/\partial \beta)(d\beta/dk_1) \\ &= ? . \end{aligned}$$

$$\begin{aligned} dB_1/dk_1 &= (\partial B_1/\partial k_1) + (\partial B_1/\partial \alpha_1)(d\alpha_1/dk_1) + (\partial B_1/\partial \alpha_2)(d\alpha_2/dk_1) + (\partial B_1/\partial \beta)(d\beta/dk_1) \\ &= ? . \end{aligned}$$

$$\begin{aligned} dC_1/dk_1 &= (\partial C_1/\partial p_c) + (\partial C_1/\partial \alpha_1)(d\alpha_1/dk_1) + (\partial C_1/\partial \alpha_2)(d\alpha_2/dk_1) + (\partial C_1/\partial \beta)(d\beta/dk_1) \\ &= ? . \end{aligned}$$

$$\begin{aligned} dE(S)/dk_1 &= (\partial E(S)/\partial k_1) + (\partial E(S)/\partial \alpha_1)(d\alpha_1/dk_1) + (\partial E(S)/\partial \alpha_2)(d\alpha_2/dk_1) \\ &\quad + (\partial E(S)/\partial \beta)(d\beta/dk_1) = ? . \end{aligned}$$

$$\begin{aligned} dA_2/dk_1 &= (\partial A_2/\partial k_1) + (\partial A_2/\partial \alpha_1)(d\alpha_1/dk_1) + (\partial A_2/\partial \alpha_2)(d\alpha_2/dk_1) + (\partial A_2/\partial \beta)(d\beta/dk_1) \\ &> 0 . \end{aligned}$$

$$\begin{aligned} dB_2/dk_1 &= (\partial B_2/\partial k_1) + (\partial B_2/\partial \alpha_1)(d\alpha_1/dk_1) + (\partial B_2/\partial \alpha_2)(d\alpha_2/dk_1) + (\partial B_2/\partial \beta)(d\beta/dk_1) \\ &= 0 . \end{aligned}$$

$$\begin{aligned} dC_2/dk_1 &= (\partial C_2/\partial k_1) + (\partial C_2/\partial \alpha_1)(d\alpha_1/dk_1) + (\partial C_2/\partial \alpha_2)(d\alpha_2/dk_1) + (\partial C_2/\partial \beta)(d\beta/dk_1) \\ &= 0 . \end{aligned}$$

$$\begin{aligned} dE(M)/dk_1 &= (\partial E(M)/\partial k_1) + (\partial E(M)/\partial \alpha_1)(d\alpha_1/dk_1) + (\partial E(M)/\partial \alpha_2)(d\alpha_2/dk_1) \\ &\quad + (\partial E(M)/\partial \beta)(d\beta/dk_1) = 0 . \end{aligned}$$

$$\begin{aligned} dE(\pi)/dk_1 &= \partial E(\pi)/\partial k_1 \\ &= -W_1' \alpha_1 ((W_1 + H^0)/3)^2 - (1 - \rho^2) \{ \alpha_1 W_1' (W_1'/3)^2 (k_1^2 + k_2^2 - 2\rho k_1 k_2) \\ &\quad + 2((W_1 + H^0)W_1')/9 \} \alpha_1 W_1'' (k_1^2 + k_2^2 - 2\rho k_1 k_2) + (W_1 + H^0)(W_1'/3)^2 (2k_1 - 2\rho k_2) \\ &< 0 . \end{aligned}$$

Q.E.D.

Remark. When the worker treats the new responsibility as a burden rather than enlarged interest, his efforts decrease. Although only one of the factors changes, both production and quality-enhancing efforts decrease due to the negative correlation between the two states of nature.

### Summary

In this chapter, a hierarchical agency model was proposed to simulate the integration of quality assurance and the production process as observed in a JIT environment. The bottom-line worker is assigned dual responsibilities. The worker is better off with this arrangement, not only through the increased compensation, but also through the enlarged job interest. The results obtained in this model are quite consistent with that in Holmstrom (1979), Basu et al. (1985) and Suh (1988), although different settings and assumptions are considered.

Both general contracts structure and more specific comparative statics were discussed in the chapter. The first-best solution cannot be reached because of the agents' moral hazard problems. Basically, the contract is an increasing function of its attributes, and the mean and variance of the attributes provide a common standard to evaluate the final performance. Several assumptions were made in an effort to derive the comparative statics results. This is a general restriction for agency theory. As Grossman and Hart (1983) point out, even the simplest results require certain restrictive assumptions. On the whole, the principal is better off to reduce the uncertainty caused by the negative correlation between quantity and quality. The principal also prefers the worker who is more interested in enlarged job assignments. When alternative job opportunities for the agents become more attractive, it is suggested that the principal should modify the compensation plan with an increased salary and lowered commission rates. Improvements in the marginal production cost and the quality mark-up price are also

encouraged by the principal. Extensions and further research of this model are discussed in chapter 6.

## CHAPTER 4

### PROFIT SHARING AND TARGET SETTING IN PROCUREMENT MANAGEMENT

A clear understanding of the relationship between procurement and production management is critical in today's business environment. Procurement management is all of the management functions related to the acquisition process, which includes deciding which vendors to use, negotiating contracts, expediting delivery, and acting as liaison between vendors and other company departments; it is not limited to placing and tracking orders. Procurement is of strategic importance and must satisfy the company's long-term supply needs to support the company's production capabilities. Recently that importance is increasing because of two factors: the tremendous impact of materials costs and quality on profits, and the increasing prominence of automated manufacturing.

The costs of purchased materials are substantial and growing. The typical U.S. manufacturer spent 40 percent of its total income from sales on purchased materials in 1945. The proportion rose to 50 percent in 1960, 60 percent in late 1980, and is expected by top executives to climb even higher (Miller and Roth, 1988). The proportion spent on purchased materials varies from industry to industry. For example, automobile manufacturers spend about 60 percent of their revenues on material purchases, farm implement manufacturers about 65 percent, food processors about 70 percent, and petroleum refining firms over 80

percent, yet the pharmaceutical industry spends only 25 percent (Gaither, 1990, p. 587). There are also some variations by country. Japanese firms, as an example, spend on average 7 percent more of their income on materials than do firms in North America and Europe due to the lack of natural resources (Krajewski and Ritzman, 1990, p. 394). Despite such variations, most firms fall within the 45 to 65 percent range, giving materials procurement great cost-reduction potential.

As the automation of manufacturing continues today, labor costs represent only about 10 to 15 percent of production costs in many mass production industries. Some estimate that labor costs will decline to about 5 percent of production costs by the year 2000 (Gaither, 1990, p. 587). Therefore, labor costs will become less significant and material costs will become the central focus in the control of production costs in some industries. Automation, especially geared toward Just-In-Time production, requires rigid control of design, delivery schedules, and quality of purchased materials. Hence, procurement management is crucial to establish and maintain vendor relations ensuring that materials of right design and of good quality are delivered in the right quantities at the right times.

Because of the significant amount of capital tied up with materials, it is particularly important to control production costs. Japanese industries, as an example, import most of their materials, yet still make considerable profits and possess large market share. This case deserves our consideration. In this chapter, a hierarchical agency model is proposed to discuss the interaction of procurement and production departments under target-setting and profit-sharing

compensation plans. It is shown that carefully designed compensation schemes can be used as a tool to provide motivation to reduce production costs.

Profit-sharing plans, in which an individual's compensation is tied to the overall performance of the firm, have received wide acceptance. Kruse (1987) reports that twenty percent of the United States labor force, about 22 million employees, participate in over 400,000 workplace profit-sharing plans, and that the number of profit-sharing pension plans has increased by 19,000 per year since 1970. A recent New York Exchange Survey indicates that 70 percent of firms with profit-sharing plans report that they lead to improved productivity (Baker, et al., 1988). Schuster's (1984) study shows that the success of a profit-sharing plan usually generates a productivity improvement of 5 to 15 percent in the first year, better work attitudes, product design improvements, and improved product quality. From the workers' viewpoint, there is greater employment stability in a difficult and turbulent environment.

The Scanlon plan is one of the best-known profit-sharing examples. First developed in the 1950s by Joe Scanlon, an ex-steelworker and steelworkers' union official who was later lured to the M.I.T. faculty, the Scanlon plan involves a combination of group and organization-level pay incentives, an employee suggestion system, and a participative approach to accessing and evaluating suggestions (Kanten, 1987; Arnold and Feldman, 1986). Production committees are set up for each department in the organization. These committees consist of the supervisor or senior manager in the department and representatives of

employees, who may either be elected or appointed by the union. These production committees screen suggestions for productivity improvement from both employees and managers. The cost savings generated by suggestions that are accepted are paid to everyone in the department originating the suggestion. In addition, under the Scanlon plan, gains resulting from productivity improvements are paid in the form of a monthly bonus to all employees. Everyone receives a share in proportion to his wage or salary level. Overall, the record of the Scanlon plan is quite positive. The most frequent benefits associated with the plan are greater organizational efficiency, increased participation in decision making, a greater willingness on the part of employees to accept change, and an improved climate of union-management relations (White, 1979).

Weitzman (1980), MacDonald (1984) and others argue that the so-called "optimal compensation schemes" of the agency problem are at a rather high level of abstraction, and provide little in the way of operational hypotheses. Therefore, the analysis of this chapter is based on a linear profit-sharing plan similar to the Scanlon plan, with a target or standard of the level of production costs that must be attained before the agents receive any share of profits. Linear contracts introduce sufficient simplification to provide interesting results without abstracting beyond reality. The linear contract has been in wide-spread use in practice (Weitzman, 1980; Cummins, 1977; Atkinson, 1978; Berhold, 1971; Schern, 1964). Atkinson (1978) prefers a procedure of standard-setting in conjunction with a profit-sharing plan as opposed to a pure profit-sharing plan. His reasons are as follows. First, the standard-setting incentive structure allows for direct

incorporation of the principal's beliefs and thus provides a specific basis for reward and control purposes. Second, under the standard-setting plan, the contractual risk for the agent is based on the effect of the agent's marginal return and not on the entire risk of the agency, hence the agent's risk will probably be lessened.

In what follows in this chapter, the next section introduces the hierarchical agency model to be utilized in this presentation. It will follow the Scanlon/Atkinson concept. Then, the characterization of the first-best and the second-best profit-sharing compensation plans are discussed and compared. Detailed comparative statics of exogenous parameters are analyzed. The chapter then concludes with a summary of the findings.

### The Model

The interaction between the principal and the agents is viewed as a game where the principal declares linear profit-sharing contracts with target costs levels to the procurement and production managers (the agents). The procurement manager and the production manager, based on the compensation plans, decide on the effort levels they plan to devote to quality materials procurement and production costs reduction, respectively, and then report their achievements. In this model both production costs ( $C$ ) and material quality ( $q$ ) are observable by three parties. This information is called "hard" which means that the agents' reports are verifiable by the principal, and false reports are not possible (Tirole, 1986). The agents must convey their achievement in a



credible way. For example, the principal can look at the sampling result of an incoming lot and the receipts of various costs during production to convince himself that the managers have truly accomplished the jobs that they declare to have accomplished.

Production costs ( $C$ ) are influenced by the production manager's effort ( $\alpha$ ), the material quality level ( $q$ ), and a random environmental factor such as machine breakdowns. Production costs can be reduced when the production manager expends more effort or the materials quality is improved. Material quality ( $q$ ) is assumed to have only two levels, high or low. For example, given an acceptable quality level (AQL) at 5 percent, a lot that has only a .5 percent defective rate can be considered to be at a high level, and a lot that has exactly a 5 percent defective rate can be considered to be at a low level. Material quality can also be defined as the combination of defective rate and delivery lead times. The chance of getting high-quality materials ( $P_h(\beta)$ ) is increasing in the procurement manager's effort  $\beta$ . It is further assumed that the production cost  $C$  is normally distributed with mean  $u(C)$  and variance  $\sigma_c^2$ , and  $u(C)=c-c_1\alpha-c_2q$  where  $c$  is the average production cost without the managers' efforts,  $c_1$  is the marginal cost reduction by the production manager's effort, and  $c_2$  is the marginal cost reduction by improving materials quality.

The benefits of other economic activities of the principal (in terms of profits) are denoted by a random variable  $R$ . For example,  $R$  can be the returns on investment in conducting research and development or in training employees. It is assumed that  $R$  is normally distributed with mean  $u(R)$  and variance  $\sigma_r^2$ , and  $u(R)=r-r_1q$  where  $r$  is the return

rate when materials quality is at the minimum acceptable level, and  $r_1$  is the marginal decrease of return due to the additional capital spent on purchasing quality materials. In general, higher quality material demands a higher purchasing price. Each dollar tied up in procurement is a dollar unavailable for investment in new products, technological improvements, or other beneficial economic activities. Nevertheless, the principal is still assumed to prefer high-quality materials, i.e.  $c_2 > r_1$ . It is also assumed that  $R$  and  $C$  are negatively correlated, i.e.  $\text{Cov}(R, C) = \rho \sigma_r \sigma_c < 0$ . Increasing production costs will reduce the profit margin and thus hinder the principal investing in other activities. Callen's (1988) paper can be considered as a special case of this model when the covariance term equals zero.

The production manager's compensation  $S_q(C)$  is based on the following formula:

$$S_q(C) = A - B(C - \hat{c}_q), \quad q = h, l,$$

where  $A$  is his base salary,  $B$  is the sharing ratio ( $0 \leq B \leq 1$ ),  $C$  is the actual production cost, and  $\hat{c}_q$  is the target cost at each quality level and  $\hat{c}_h < \hat{c}_l$ . Cost saving is shared by the managers. According to the Scanlon plan, the procurement manager will receive a share in proportion to his salary level. Therefore, the procurement manager's compensation  $M_q(C) = A_2 - B_2(C - \hat{c}_q) = A_2 - kB(C - \hat{c}_q) = k(A - B(C - \hat{c}_q)) = kS_q(C)$ , where  $k = A_2/A_1 \geq 0$  is called the departmental differentiation factor.

The production manager's utility,  $U(S_q(C) - d_1(\alpha))$ , is defined over his payment and his personal effort; and  $d_1(\alpha)$  is a convex function. Similarly, the procurement manager's utility function,  $V(M_q(C) - d_2(\beta))$ , is defined over his payment and his personal effort, and  $d_2(\beta)$  is a

convex function. Both managers are risk avoiders. It is the principal who decides  $B$  and  $k$ . The production level is assumed at a fixed volume and price, therefore, its revenue can be dropped out from the principal's profit function. The principal's utility function,  $I(\pi)$ , is defined over his profit  $\pi$ , where  $\pi = R - M_q(C) - S_q(C) - C = R - (k+1)S_q(C) - C$ . Thus, the agency model can be established as follows. The principal seeks to

$$\text{MAX}_{B, k} \quad E[I(\pi)] = \sum_q P_q(\beta) I[R - M_q(C) - S_q(C) - C] \quad (4.1)$$

$$\text{Subject to} \quad \sum_q P_q(\beta) U(S_q(C) - d_1(\alpha)) \geq U(S^\circ) \quad (4.2)$$

$$\text{Max}_{\alpha} \quad \sum_q P_q(\beta) U(S_q(C) - d_1(\alpha)) \quad (4.3)$$

$$\sum_q P_q(\beta) V(M_q(C) - d_2(\beta)) \geq V(M^\circ) \quad (4.4)$$

$$\text{Max}_{\beta} \quad \sum_q P_q(\beta) V(M_q(C) - d_2(\beta)) \quad (4.5)$$

$$\sum_q P_q(\beta) = 1,$$

where  $S^\circ$  and  $M^\circ$  are the managers' reservation welfare levels.

In addition, the principal and the managers are assumed to have exponential utility functions that exhibit constant absolute risk aversion (CARA). Let  $n$ ,  $n_1$ , and  $n_2$  be the Arrow-Pratt measure of absolute risk aversion for the principal, the production manager, and the procurement manager, respectively. As is well known, the normally distributed random variable permits one to employ a mean-variance model (Tobin, 1958; Feldstein, 1969; Bamberg, 1986; Epps, 1981). The popularity of the mean-variance approach is due to the property that risk is fully described by the variance of the random variable. Thus,

along with the CARA utility function, the principal's objective can be written as

$$\begin{aligned} \text{MAX}_{B,k} \quad & \Sigma P_q(\beta) \{ (r-r_1q) - (k+1)A + (k+1)B[c - c_1\alpha - c_2q - \hat{c}_q] - (c - c_1\alpha - c_2q) \} \\ & - (n/2) \{ \sigma_r^2 - 2[1 - (k+1)B] \rho \sigma_r \sigma_c + [1 - (k+1)B]^2 \sigma_c^2 \}. \end{aligned}$$

The maximization of the two managers' expected utilities is also identical to maximization of the associated certainty equivalents as follows:

$$\begin{aligned} \text{MAX}_{\alpha} \quad & \Sigma_q P_q [A - B(c - c_1\alpha - c_2q - \hat{c}_q)] - d_1(\alpha) - (n_1/2) B^2 \sigma_c^2, \\ \text{and } \text{MAX}_{\beta} \quad & \Sigma_q P_q k [A - B(c - c_1\alpha - c_2q - \hat{c}_q)] - d_2(\beta) - (n_2/2) k^2 B^2 \sigma_c^2. \end{aligned}$$

Substituting (4.3) and (4.5) with their corresponding first-order conditions, the above agency model can be written as

[MOD]

$$\begin{aligned} \text{MAX}_{B,k} \quad & \Sigma_q P_q(\beta) \{ (r-r_1q) - (k+1)A + (k+1)B[c - c_1\alpha - c_2q - \hat{c}_q] - (c - c_1\alpha - c_2q) \} \\ & - (n/2) \{ \sigma_r^2 - 2[1 - (k+1)B] \rho \sigma_r \sigma_c + [1 - (k+1)B]^2 \sigma_c^2 \} \end{aligned} \quad (4.6)$$

Subject to

$$\Sigma_q P_q [A - B(c - c_1\alpha - c_2q - \hat{c}_q)] - d_1(\alpha) - (n_1/2) B^2 \sigma_c^2 \geq S^0 \quad (4.7)$$

$$BC_1 - d_1'(\alpha) = 0 \quad (4.8)$$

$$\Sigma_q P_q k [A - B(c - c_1\alpha - c_2q - \hat{c}_q)] - d_2(\beta) - (n_2/2) k^2 B^2 \sigma_c^2 \geq M^0 \quad (4.9)$$

$$BkP_h'(\beta) [c_2(h-1) + \hat{c}_h - \hat{c}_1] - d_2'(\beta) = 0 \quad (4.10)$$

$$\Sigma P_q(\beta) = 1, \quad q=h, 1$$

$$0 \leq (k+1)B \leq 1, \text{ and } k \geq 0.$$

### Characterization of Optimal Compensation

In the absence of constraints (4.8) and (4.10), the above [MOD] program determines the first-best solution which serves as a benchmark to be compared with the second-best solution.

#### The First-Best Solution

The first-best solution implies that the principal could costlessly monitor the managers' effort levels or decisions. For example, in a highly-centralized firm, the managers have to choose the effort levels that maximize the principal's objective rather than maximize their own objectives. Consequently, the principal has no incentive to offer more than each manager's reservation welfare. Constraints (4.7) and (4.9) should be solved as equalities:

$$\Sigma P_q[A-B(c-c_1\alpha-c_2q-\hat{c}_q)] = d_1(\alpha)+(n_1/2)B^2\sigma_c^2+S^\circ \quad (4.11)$$

$$\Sigma P_qk[A-B(c-c_1\alpha-c_2q-\hat{c}_q)] = d_2(\beta)+(n_2/2)k^2B^2\sigma_c^2+M^\circ. \quad (4.12)$$

substituting equalities (4.11) and (4.12) into (4.6) yields

$$\begin{aligned} \text{MAX}_{B,k,\alpha,\beta} \quad & \{ (r-r_1l) - (c-c_1\alpha-c_2l) + P_h(c_2-r_1)(h-l) - d_1(\alpha) - d_2(\beta) \\ & - (1/2)B^2\sigma_c^2(n_1+n_2k^2) - S^\circ - M^\circ \} \\ & - (n/2)\{\sigma_r^2 - 2[1-(k+1)B]\rho\sigma_r\sigma_c + [1-(k+1)B]^2\sigma_c^2\}, \end{aligned} \quad (4.13)$$

which is a concave function given nonnegative degrees of risk aversion.

A contract  $(A, B, k, \hat{c}_q)$  is first-best optimal if and only if  $(B, k)_{FB}$  maximizes (4.13), subject to the restrictions  $k \geq 0$  and  $0 \leq (k+1)B \leq 1$ .

The first-order conditions of (4.13) with respect to  $k$  and  $B$  are:

$$I_k = B\{B\sigma_c^2(n_2k+n(k+1)) - n(\sigma_c^2 - \rho\sigma_r\sigma_c)\} = 0 \quad (4.14)$$

$$\text{and } I_B = B\sigma_c^2(n_1+n_2k^2+n(k+1)^2)-n(k+1)(\sigma_c^2-\rho\sigma_r\sigma_c) = 0. \quad (4.15)$$

Solving (4.14) and (4.15) simultaneously yields

$$k_{fb} = n_1/n_2 \quad (4.16)$$

$$\text{and } B_{fb} = nn_2z/(n_1n_2+nn_1+nn_2)\sigma_c^2 \quad (4.17)$$

where  $z = \sigma_c^2 - \rho\sigma_r\sigma_c$ .

Therefore, the procurement manager's optimal sharing ratio  $(kB)_{fb}$ , and the first-best sharing pool  $((k+1)B)_{fb}$  (i.e. the sum of the two managers' sharing ratios) are:

$$(kB)_{fb} = nn_1z/(n_1n_2+nn_1+nn_2)\sigma_c^2.$$

$$\text{and } ((k+1)B)_{fb} = n(n_1+n_2)z/(n_1n_2+nn_1+nn_2)\sigma_c^2 \quad (4.18)$$

In the first-best case, the principal has centralized control power. By maximizing the principal's objective function (4.13) with respect to  $\alpha$  and  $\beta$ , the managers' effort levels are assigned to satisfy the following conditions:

$$I_\alpha = c_1 - d_1'(\alpha) = 0, \text{ and} \quad (4.19)$$

$$I_\beta = P_h'(\beta)[(c_2 - r_1)(h-1)] - d_2'(\beta) = 0. \quad (4.20)$$

From (4.19), the production manager's marginal expenditure of effort equals the marginal contribution to cost reduction. From (4.20), the procurement manager's marginal expenditure of effort equals the marginal changes to the principal's profits.

The characterization of the first-best solution is discussed in the following results.

Result 4.1. The optimal first-best departmental differentiation factor,  $k_{fb}$ , depends only on the managers' degree of risk aversion.

Proof. This result follows directly from examining equation (4.16).

Q.E.D.

Remark. When the principal can monitor each manager's decision choice (effort) perfectly, the departmental differentiation factor does not depend on the nature of responsibilities at each position. Therefore, no matter which job is assigned, each manager is paid the same contract. This is more likely in a highly-centralized firm such as exists in a socialist society. In practice, however, payment is usually tied to one's position in the firm. Medoff and Abraham (1980), for example, find that between-job-level earnings differentials are more important than within-job-level differentials. Also, Murphy (1985) finds that corporate vice presidents receive average pay increases of 18.8 percent upon promotion to another vice-presidential or higher position, compared to average pay increases of only 3.3 percent in years when they remain in the same position. This phenomenon is explained later in the second-best case.

Result 4.2. Both managers' optimal first-best sharing ratios,  $B_{fb}$  and  $(kB)_{fb}$ , and the sharing pool  $((k+1)B)_{fb}$  are:

- (1) between zero and one, and in total less than one;
- (2) independent of each manager's efforts;
- (3) decreasing in the manager's own degree of risk aversion;
- (4) increasing when there is more negative correlation between  $R$  and  $C$ ;
- (5) decreasing in the volatility of production processes;
- (6) increasing in the degree of risk aversion of the principal. And each manager's optimal sharing ratio increases when the other manager becomes more risk averse.

Proof. (1): From (4.17) and (4.18), unless the covariance is "much larger" than the variance of costs,  $B_{fb}$ ,  $(kB)_{fb}$  and  $((k+1)B)_{fb}$  are likely to be between zero and one.

(2): The sharing ratios are independent of managers' efforts since the effort levels are determined by the principal directly from (4.19) and (4.20).

(3)-(6): From (4.19) and (4.20),  $I_\alpha$  and  $I_\beta$  are not functions of  $B$ ,  $k$ ,  $n$ ,  $n_1$ ,  $n_2$ , and  $\sigma_c^2$ . Therefore, only  $I_k$  and  $I_B$  need to be considered. Taking total derivatives of  $I_k$  and  $I_B$  with respect to a parameter  $t$  yields

$$\begin{aligned} & (\partial I_k / \partial t) + (\partial I_k / \partial B) (dB/dt) + (\partial I_k / \partial k) (dk/dt) + (\partial I_k / \partial \alpha) (d\alpha/dt) + (\partial I_k / \partial \beta) (d\beta/dt) \\ &= (\partial I_k / \partial t) + [\sigma_c^2 (n_2 k + n(k+1))] (dB/dt) + [B \sigma_c^2 (n + n_2)] (dk/dt) = 0, \text{ and} \end{aligned} \quad (4.21)$$

$$\begin{aligned} & (\partial I_B / \partial t) + (\partial I_B / \partial B) (dB/dt) + (\partial I_B / \partial k) (dk/dt) + (\partial I_B / \partial \alpha) (d\alpha/dt) + (\partial I_B / \partial \beta) (d\beta/dt) \\ &= (\partial I_B / \partial t) + \sigma_c^2 [n_1 + n_2 k^2 + n(k+1)^2] (dB/dt) + [2B \sigma_c^2 (n_2 k + n(k+1)) - n_2] (dk/dt) \\ &= 0. \end{aligned} \quad (4.22)$$

Solving (4.21) and (4.22) simultaneously,

$$dB/dt = -J((I_k, I_B)/(t, k)) / J((I_k, I_B)/(B, k)), \text{ and}$$

$$dk/dt = -J((I_k, I_B)/(B, t)) / J((I_k, I_B)/(B, k)),$$

where  $J(\cdot)$  is the notation of the Jacobian.

It is straight forward to see that  $J((I_k, I_B)/(B, k)) = -nn_2\sigma_c^2 z < 0$ .

Therefore,  $\text{sign}[dB_{fb}/dt] = \text{sign}[J((I_k, I_B)/(t, k))]$ ,

$$\text{sign}[dk_{fb}/dt] = \text{sign}[J((I_k, I_B)/(B, t))],$$

$$\text{sign}[d(kB)_{fb}/dt] = \text{sign}[k dB_{fb}/dt + B dk_{fb}/dt], \text{ and}$$

$$\text{sign}[d((k+1)B)_{fb}/dt] = \text{sign}[d(kB)_{fb}/dt + dB_{fb}/dt].$$

Hence, results (3)-(6) are obtained by examining the following signs:

$$\text{sign}[dB_{fb}/dn] = \text{sign}[(k/n)(n_2 B \sigma_c^2)^2] > 0.$$



$$\text{sign} [dB_{fb}/dn_1] = \text{sign} [-(n+n_2)(B\sigma_c^2)^2] < 0.$$

$$\text{sign} [dB_{fb}/dn_2] = \text{sign} [nk(\sigma_c^2)^2] > 0.$$

$$\text{sign} [dB_{fb}/d\sigma_c^2] = \text{sign} [-nn_2B\sigma_c^2] < 0.$$

$$\text{sign} [dB_{fb}/d|\text{Cov}(R,C)|] = \text{sign} [nn_2B\sigma_c^2] > 0.$$

$$\text{sign} [d(kB)_{fb}/dn] = \text{sign} [(k^2/n)(n_2B\sigma_c^2)^2] > 0.$$

$$\text{sign} [d(kB)_{fb}/dn_1] = \text{sign} [n(\sigma_c^2)^2] > 0.$$

$$\text{sign} [d(kB)_{fb}/dn_2] = \text{sign} [-(n+n_1)k(\sigma_c^2)^2] < 0.$$

$$\text{sign} [d(kB)_{fb}/d\sigma_c^2] = \text{sign} [-knn_2B\sigma_c^2] < 0.$$

$$\text{sign} [d(kB)_{fb}/d|\text{Cov}(R,C)|] = \text{sign} [nn_2Bk\sigma_c^2] > 0.$$

$$\text{sign} [d((k+1)B)_{fb}/dn] > 0.$$

$$\text{sign} [d((k+1)B)_{fb}/dn_1] = \text{sign} [-n_2(\sigma_c^2)^2] < 0.$$

$$\text{sign} [d((k+1)B)_{fb}/dn_2] = \text{sign} [-n_1k(\sigma_c^2)^2] < 0.$$

$$\text{sign} [d((k+1)B)_{fb}/d\sigma_c^2] < 0.$$

$$\text{sign} [d((k+1)B)_{fb}/d|\text{Cov}(R,C)|] > 0.$$

Q.E.D.

Result 4.3. If the principal is risk neutral, then he bears all risk, and the managers are paid a fixed salary.

Proof. Result 4.3 follows directly from examining equations (4.14) and (4.15).

Q.E.D.

### The Second-Best Solution

The managers would maximize their own objectives rather than follow the principal's instructions when their decisions (efforts) are not observable by the principal. The second-best optimal compensations can be obtained by first solving (4.8) and (4.10) for the optimal effort levels as implicit functions of the sharing ratios and the departmental differentiation factor, i.e.  $\alpha=\alpha(B,k)$  and  $\beta=\beta(B,k)$ . Then, solve the

following function by substituting  $\alpha(B,k)$  and  $\beta(B,k)$  along with (4.7) and (4.9) into equation (4.6).

$$\begin{aligned} \text{MAX} \quad & (r-r_1) - (c-c_1\alpha(B,k) - c_2l) + P_h(c_2-r_1)(h-1) - d_1(\alpha(B,k)) - d_2(\beta(B,k)) \\ & - (1/2)B^2\sigma_c^2(n_1+n_2k^2) - S^\circ - M^\circ - (n/2)\{\sigma_r^2 - 2[1-(k+1)B]\rho\sigma_r\sigma_c + [1-(k+1)B]^2\sigma_c^2\}, \end{aligned} \quad (4.23)$$

which is a concave function of  $B$  and  $k$ , given nonnegative degrees of risk aversion. A contract  $(A, B, k, \hat{c}_q)$  is second-best optimal if and only if  $(B, k)_{sb}$  maximizes (4.23), subject to the restriction  $k \geq 0$  and  $0 \leq (k+1)B \leq 1$ .

The first-order conditions of (4.23) with respect to  $k$  and  $B$  are:

$$\begin{aligned} [c_1 - d_1'(\alpha)](\partial\alpha/\partial k) + [P_h'(c_2-r_1)(h-1) - d_2'(\beta)](\partial\beta/\partial k) + Bn(\sigma_c^2 - \rho\sigma_r\sigma_c) \\ - B^2\sigma_c^2(n_2k+n(k+1)) = 0, \text{ and} \end{aligned} \quad (4.24)$$

$$\begin{aligned} [c_1 - d_1'(\alpha)](\partial\alpha/\partial B) + [P_h'(c_2-r_1)(h-1)P_h'(\beta) - d_2'(\beta)](\partial\beta/\partial B) \\ + n(k+1)(\sigma_c^2 - \rho\sigma_r\sigma_c) - B\sigma_c^2(n_1+n_2k^2+n(k+1)^2) = 0. \end{aligned} \quad (4.25)$$

Implicit differentiation of equations (4.8) and (4.10) yields:

$$\partial\alpha/\partial k = 0 \quad (4.26)$$

$$\partial\alpha/\partial B = c_1/d_1''(\alpha) > 0 \quad (4.27)$$

$$\partial\beta/\partial k = -\{P_h'B[c_2(h-1)+\hat{c}_h-\hat{c}_1]\}/\{P_h''Bk[c_2(h-1)+\hat{c}_h-\hat{c}_1]-d_2''\} > 0 \quad (4.28)$$

$$\partial\beta/\partial B = -\{P_h'k[c_2(h-1)+\hat{c}_h-\hat{c}_1]\}/\{P_h''Bk[c_2(h-1)+\hat{c}_h-\hat{c}_1]-d_2''\} > 0 \quad (4.29)$$

With no loss of generality, let  $P_h''(\beta)$  equal zero. Substituting (4.26)-(4.29) into equations (4.24) and (4.25) yields:

$$L_k = a_2 + n_2z - (n_2k + n(k+1))B\sigma_c^2 = 0, \quad (4.30)$$

$$\text{and } L_B = a_1 + ka_2 + n(k+1)z - (n_1 + n_2k^2 + n(k+1)^2)B\sigma_c^2 = 0, \quad (4.31)$$

$$\text{where } a_1 = c_1(c_1 - d_1')/d_1'', \quad (4.32)$$

$$a_2 = P_h'xy/d_2'', \quad (4.33)$$

$$x = c_2(h-1) + \hat{c}_h - \hat{c}_1, \text{ and } y = (c_2 - r_1)(h-1)P_h' - d_2'.$$

Solving (4.30) and (4.31) simultaneously yields:

$$k_{sb} = [(n+n_1)a_2 - na_1 + nn_1z] / [(n+n_2)a_1 - na_2 + nn_2z],$$

$$\text{if this expression} \geq 0, \quad (4.34)$$

$$= 0, \quad \text{if the above expression is} < 0.$$

$$B_{sb} = [(n+n_2)a_1 - na_2 + nn_2z] / (n_1n_2 + nn_1 + nn_2)\sigma_c^2,$$

$$\text{if this expression is in } [0,1], \quad (4.35)$$

$$= 1, \quad \text{if the above expression is} > 1,$$

$$= 0, \quad \text{if the above expression is} < 0.$$

Recalling from (4.8) and (4.10), the managers' effort levels are selected as follows:

$$L_\alpha = Bc_1 - d_1'(\alpha) = 0, \text{ and} \quad (4.8)$$

$$L_\beta = BkP_h'x - d_2'(\beta) = 0. \quad (4.10)$$

Suppose that interior solutions exist, some results regarding the characteristics of the second-best solution are discussed as follows.

Result 4.4. The optimal second-best departmental differentiation factor,  $k_{sb}$ , depends not only on the managers' risk attitudes, but also on each position's contribution to the overall profits.

Proof. From (4.32) and (4.33),  $a_1$  and  $a_2$  can be treated as the marginal contribution to the overall profits by increasing  $\alpha$  and  $\beta$ , respectively.

The result follows directly. Q.E.D.

Remark. When the principal cannot monitor the managers' efforts, the departmental differentiation factor is tied to the position in the hierarchy. The same person might be paid differently because of different job titles. Lack of perfect monitoring schemes can explain this common phenomenon in practice.

Result 4.5. The production manager's second-best effort level is increasing in the sharing ratio. The procurement manager's second-best effort level is increasing in the sharing ratio or the departmental differentiation factor. Each manager's second-best effort level is less than his first-best effort level (i.e.  $\alpha_{sb} < \alpha_{fb}$  and  $\beta_{sb} < \beta_{fb}$ ).

Proof. Comparing equations (4.8) with (4.19) and (4.10) with (4.20), since  $d_1(\alpha)$ ,  $d_2(\beta)$  and  $P_h(\beta)$  are increasing in  $\alpha$  or  $\beta$ , and  $B_{sb}$  is between zero and one, the result follows directly. Q.E.D.

Result 4.6. The second-best sharing pool is greater than the first-best sharing pool (i.e.  $[(k+1)B]_{sb} > [(k+1)B]_{fb}$ ). The second-best effects on separate  $k_{sb}$  and  $B_{sb}$ , however, are in the opposite direction.

Proof. From (4.18),  $[(k+1)B]_{fb} = n(n_1+n_2)z/(n_1n_2+nn_1+nn_2)\sigma_c^2$ .

From (4.34) and (4.35),

$$[(k+1)B]_{sb} = [n_1a_2+n_2a_1+n(n_1+n_2)z]/(n_1n_2+nn_1+nn_2)\sigma_c^2. \quad (4.36)$$

Therefore,  $[(k+1)B]_{sb} - [(k+1)B]_{fb} = [n_1a_2+n_2a_1]/(n_1n_2+nn_1+nn_2)\sigma_c^2 > 0$ .

From (4.16) and (4.34),  $\text{sign}[k_{sb}-k_{fb}] = \text{sign}[(n_1n_2+nn_1+nn_2)(a_2-a_1)]$ . (4.37)

From (4.17) and (4.35),  $\text{sign}[B_{sb}-B_{fb}] = \text{sign}[n_2a_1+n(a_1-a_2)]$ . (4.38)

If  $a_1-a_2 \geq 0$ , then  $B_{sb} \geq B_{fb}$  and  $k_{sb} \leq k_{fb}$ . If  $a_1-a_2 < 0$ , then most likely  $B_{sb} < B_{fb}$  and  $k_{sb} > k_{fb}$ . Q.E.D.

Remark. Results (4.5) and (4.6) reveal the important so called "agency costs," which measure the deviation from Pareto-optimal risk sharing, from the principal's point of view. Let us define agency costs (AC) as

$$AC = E[I(k_{fb}, B_{fb}) - I(k_{sb}, B_{sb})]. \quad (4.39)$$

Then, agency costs as a measure of distance can be presumed to give an estimate of how much (in terms of utility) the second-best contracts could be improved if there were monitoring of the managers' efforts. In

fact, the nature of agency costs could be treated as an expected value (in terms of utility) of perfect information. The intuition of the opposite effect on  $k$  and  $B$  separately is as follows. The comparison of  $a_1$  and  $a_2$  indicates the relative importance of these two departments. If the profit contribution from cost-reducing efforts is greater than that from quality-improving efforts (i.e.  $a_1 > a_2$ ), then the production manager should be compensated more to induce his efforts, and vice versa. By and large, the principal has to pay more from the sharing pool and the managers actually expend less effort when the principal lacks a costless monitoring mechanism.

Result 4.7. Suppose the principal is risk neutral. Unlike the first-best solution, the principal does not bear the entire contractual risk.

Proof. Given  $n=0$ ,  $n_1 > 0$  and  $n_2 > 0$ ,  $B_{fb}=0$ . At the second-best case,  $k_{sb}=n_1 a_2 / n_2 a_1 > 0$  and  $B_{sb}=a_1 / n_1 \sigma_c^2 > 0$ . The risk-averse managers still have to share some risk. Q.E.D.

Result 4.8. In both first-best and second-best cases, profit-sharing compensation is preferred to fixed salary.

Proof. From (4.18) and (4.36),  $[(k+1)B]_{fb}$  and  $[(k+1)B]_{sb}$  are both positive. Result 4.8 follows directly. Q.E.D.

Remark. This result shows that it is better for the principal to offer profit-sharing compensation rather than fixed salaries whether he can monitor the managers' efforts or not.

### The Comparative Statics

Until now this analysis has concentrated on optimal compensation for a given environment (parameters). It will be interesting to investigate the effects on contract designs of changing environments. In the following results, several factors are discussed: the effect of changing the participants' risk attitudes ( $n$ ,  $n_1$ , and  $n_2$ ), the production cost volatility ( $\sigma_c^2$ ), the covariance ( $\text{Cov}(R, C)$ ), the production manager's cost-reducing coefficient ( $c_1$ ), the procurement manager's cost-reducing coefficient ( $c_2$ ), the principal's return-reducing coefficient ( $r_1$ ), the gap between cost targets ( $\hat{c}_h - \hat{c}_1$ ), the difference between attainable material quality ( $h-1$ ), the marginal probability of getting high-quality materials ( $P_h'$ ), and the degree of effort aversion ( $-d_1''/d_1'$  and  $-d_2''/d_2'$ ) on (1) the production manager's sharing ratio  $(B_{sb})$ , (2) the procurement manager's sharing ratio  $(kB)_{sb}$ , and (3) the principal's sharing pool  $[(k+1)B]_{sb}$ . Hereafter, analysis focuses on the second-best case. The subscript (sb) has been dropped for convenience.

Taking total derivatives of  $L_k$ ,  $L_B$ ,  $L_\alpha$ , and  $L_\beta$  with respect to a parameter  $t$  yields

$$\begin{aligned}
 & (\partial L_k / \partial t) + (\partial L_k / \partial B) (dB/dt) + (\partial L_k / \partial k) (dk/dt) + (\partial L_k / \partial \alpha) (d\alpha/dt) + (\partial L_k / \partial \beta) (d\beta/dt) \\
 & = (\partial L_k / \partial t) - [\sigma_c^2 (n_2 k + n(k+1))] (dB/dt) - [B \sigma_c^2 (n + n_2)] (dk/dt) + 0 (d\alpha/dt) \\
 & - x P_h' (d\beta/dt) = 0, \tag{4.40}
 \end{aligned}$$

$$\begin{aligned}
 & (\partial L_B / \partial t) + (\partial L_B / \partial B) (dB/dt) + (\partial L_B / \partial k) (dk/dt) + (\partial L_B / \partial \alpha) (d\alpha/dt) + (\partial L_B / \partial \beta) (d\beta/dt) \\
 & = (\partial L_B / \partial t) - \sigma_c^2 [n_1 + n_2 k^2 + n(k+1)^2] (dB/dt) - [B \sigma_c^2 (n_2 k + n(k+1))] (dk/dt) \\
 & - c_1 (d\alpha/dt) - k x P_h' (d\beta/dt) = 0, \tag{4.41}
 \end{aligned}$$

$$\begin{aligned}
& (\partial L_\alpha / \partial t) + (\partial L_\alpha / \partial B) (dB/dt) + (\partial L_\alpha / \partial k) (dk/dt) + (\partial L_\alpha / \partial \alpha) (d\alpha/dt) + (\partial L_\alpha / \partial \beta) (d\beta/dt) \\
& = (\partial L_\alpha / \partial t) + c_1 (dB/dt) + 0 (dk/dt) - d_1'' (d\alpha/dt) + 0 (d\beta/dt) = 0, \text{ and} \quad (4.42)
\end{aligned}$$

$$\begin{aligned}
& (\partial L_\beta / \partial t) + (\partial L_\beta / \partial B) (dB/dt) + (\partial L_\beta / \partial k) (dk/dt) + (\partial L_\beta / \partial \alpha) (d\alpha/dt) + (\partial L_\beta / \partial \beta) (d\beta/dt) \\
& = (\partial L_\beta / \partial t) + kxP_h' (dB/dt) + BP_h' x (dk/dt) + 0 (d\alpha/dt) - d_2'' (d\beta/dt) = 0. \quad (4.43)
\end{aligned}$$

Solving equations (4.40) to (4.43) simultaneously,

$$dB/dt = -J((L_k, L_B, L_\alpha, L_\beta)/(t, k, \alpha, \beta))/J((L_k, L_B, L_\alpha, L_\beta)/(B, k, \alpha, \beta)), \text{ and}$$

$$dk/dt = -J((L_k, L_B, L_\alpha, L_\beta)/(B, t, \alpha, \beta))/J((L_k, L_B, L_\alpha, L_\beta)/(B, k, \alpha, \beta)), \text{ where } J(\cdot)$$

is the notation of the Jacobian. It is straight forward to see that

$$J((L_k, L_B, L_\alpha, L_\beta)/(B, k, \alpha, \beta))$$

$$= -Bc_1^2 [(xP_h')^2 + d_2'' \sigma_c^2 (n+n_2)] - d_1'' B \sigma_c^2 [d_2'' \sigma_c^2 (n_1 n_2 + n n_1 + n n_2) + (n+n_1) (xP_h')^2] < 0.$$

$$\text{Therefore, } \text{sign}[dB/dt] = \text{sign}[J((L_k, L_B, L_\alpha, L_\beta)/(t, k, \alpha, \beta))],$$

$$\text{sign}[dk/dt] = \text{sign}[J((L_k, L_B, L_\alpha, L_\beta)/(B, t, \alpha, \beta))],$$

$$\text{sign}[d(kB)/dt] = \text{sign}[k(dB/dt) + B(dk/dt)], \text{ and}$$

$$\text{sign}[d((k+1)B)/dt] = \text{sign}[d(kB)/dt + dB/dt].$$

#### The Effect of Participants' Risk Attitudes ( $n, n_1, n_2$ )

Result 4.9. Each manager's sharing ratio increases with an increase of his own degree of risk aversion or an decrease of the other manager's degree of risk aversion. The sharing pool decreases when the managers become more risk averse.

Proof. This result is obtained by examining the following signs.

$$\text{sign}[dB/dn_1] = \text{sign}[-B \sigma_c^2 d_1'' (d_2'' B \sigma_c^2 (n+n_2) - B(xP_h')^2)] < 0.$$

$$\text{sign}[dB/dn_2] = \text{sign}[d_1'' d_2'' n (B \sigma_c^2)^2] > 0.$$

$$\text{sign}[dB/dn] = \text{sign}[-(B \sigma_c^2 (k+1) - z)] = ?$$

$$\text{sign}[dkB/dn_1] = \text{sign}[nd_1'' d_2'' (B \sigma_c^2)^2] > 0.$$

$$\text{sign}[dkB/dn_2] = \text{sign}[-d_2'' k (B \sigma_c^2) (d_1'' \sigma_c^2 (n+n_1) + c_1^2)] < 0.$$

$$\text{sign}[dkB/dn] = \text{sign}[-(B\sigma_c^2(k+1)-z)(d_1''\sigma_c^2n_1-c_1^2)] = ?$$

$$\text{sign}[d(k+1)B/dn_1] = \text{sign}[-n_2d_1''d_2''(B\sigma_c^2)^2-d_1''(B\sigma_c^2xP_h')^2] < 0$$

$$\text{sign}[d(k+1)B/dn_2] = \text{sign}[-kd_2''(B\sigma_c^2)^2(d_1''n_1\sigma_c^2+c_1^2)] < 0$$

$$\text{sign}[d(k+1)B/dn] = \text{sign}[-(B\sigma_c^2(k+1)-z)B(d_1''(xP_h')^2+d_1''d_2''\sigma_c^2(n_1+n_2)-c_1^2d_2'')] = ?$$

Q.E.D.

The Effect of Volatility of Production Processes ( $\sigma_c^2$ ) and Covariance (Cov(R,C))

Result 4.10. Each manager's sharing ratio and the sharing pool increase when R and C are more negatively correlated. The effect of increasing volatility of production depends on the value of  $a_1$  and  $a_2$ . If the marginal profit of the production manager's effort is higher than the procurement manager's effort, the sharing ratios and sharing pool decrease with an increase of production risk.

Proof. This result is obtained by examining the following signs.

$$\text{sign}[dB/d\text{Cov}(R,C)] = \text{sign}[d_1''nB(d_2''n_2\sigma_c^2+(xP_h')^2)] > 0$$

$$\text{sign}[dkB/d\text{Cov}(R,C)] = \text{sign}[d_1''d_2''nn_1B\sigma_c^2+d_2''c_1^2] > 0.$$

$$\text{sign}[d(k+1)B/d\text{Cov}(R,C)] > 0.$$

$$\text{sign}[dB/d\sigma_c^2] = \text{sign}[-d_1''B(1/\sigma_c)(-2a_1+n\rho\sigma_r\sigma_c)(-d_2''\sigma_c^2n_2-(xP_h')^2)+2d_2''nB\sigma_c(a_1-a_2)] < 0 \text{ if } a_1 > a_2.$$

$$\text{sign}[dkB/d\sigma_c^2] = \text{sign}[(1/\sigma_c)(-2a_1+n\rho\sigma_r\sigma_c)(d_1''Bk(d_2''\sigma_c^2n_2+(xP_h')^2)-kd_1''(xP_h')^2)-2d_1''d_2''nBk\sigma_c(a_1-a_2)+(1/\sigma_c)(-2a_2+\rho\sigma_r\sigma_c)(c_1^2d_2'')] < 0 \text{ most likely.}$$

$$\text{sign}[d(k+1)B/d\sigma_c^2]$$

$$= \text{sign}[(1/\sigma_c)(-2a_1+n\rho\sigma_r\sigma_c)(d_1''B(k+1)(d_2''\sigma_c^2n_2+(xP_h')^2)-kd_1''(xP_h')^2)-$$

$$2d_1''d_2''nB(k+1)\sigma_c(a_1-a_2)+(1/\sigma_c)(-2a_2+\rho\sigma_r\sigma_c)(c_1^2d_2'')] < 0 \text{ if } a_1 > a_2. \quad \text{Q.E.D.}$$



Recalling from (4.32) and (4.33),  $a_1=c_1(c_1-d_1')/d_1''$ ,  $a_2=P_h'xy/d_2''$ ,  
 $x= c_2(h-1)+\hat{c}_h-\hat{c}_1$ , and  $y= (c_2-r_1)(h-1)P_h'-d_2'$ .

The Effect of The Production Manager's Cost-Reducing Coefficient ( $c_1$ ),  
 The Procurement Manager's Cost-Reducing Coefficient ( $c_2$ ), and The  
 Principal's Return-Reducing Coefficient ( $r_1$ )

Result 4.11. The production (procurement) manager's sharing ratio increases (decreases) with an increase of  $c_1$ ; decreases (increases) with an increase of  $c_2$ ; and increases (decreases) with an increase of  $r_1$ . The sharing pool increases with an increase of  $c_1$  or  $c_2$ ; decreases with an increase of  $r_1$ .

Proof. Result 4.11 follows by examining the following signs.

$$\text{sign}[dB/dc_1] = \text{sign}[2B(c_1-d_1')(d_2''\sigma_c^2(n+n_2)+(xP_h')^2)] > 0.$$

$$\text{sign}[dkB/dc_1] = \text{sign}[-2Bnd_2''\sigma_c^2(c_1-d_1')] < 0.$$

$$\text{sign}[d(k+1)B/dc_1] = \text{sign}[2B(n_2d_2''\sigma_c^2+(xP_h')^2)(c_1-d_1')] > 0.$$

$$\text{sign}[dB/dc_2] = \text{sign}[-d_1''d_2''B\sigma_c^2n(\partial a_2/\partial c_2)] < 0.$$

$$\text{sign}[dkB/dc_2] = \text{sign}[(d_1''d_2''\sigma_c^2B(n+n_1)+Bc_1^2d_2'')(\partial a_2/\partial c_2)] > 0.$$

$$\text{sign}[d(k+1)B/dc_2] = \text{sign}[(\partial a_2/\partial c_2)(d_1''d_2''\sigma_c^2Bn_1+Bc_1^2d_2'')] > 0.$$

$$\text{sign}[dB/dr_1] = \text{sign}[(\partial a_2/\partial r_1)(-d_1''d_2''B\sigma_c^2n)] > 0.$$

$$\text{sign}[dkB/dr_1] = \text{sign}[(\partial a_2/\partial r_1)(d_1''d_2''B\sigma_c^2(n+n_1)+Bc_1^2d_2'')] < 0.$$

$$\text{sign}[d(k+1)B/dr_1] = \text{sign}[(\partial a_2/\partial r_1)(d_1''d_2''B\sigma_c^2n_1+Bc_1^2d_2'')] < 0. \quad \text{Q.E.D.}$$

The Effect of The Gap of Target Costs ( $\hat{c}_h-\hat{c}_1$ )

Result 4.12. The production (procurement) manager's sharing ratio increases (decreases) with an increase of the gap of target costs. The overall effect decreases the sharing pool.

Proof. Result 4.12 follows by examining the following signs.

$$\text{sign}[dB/d|\hat{c}_h - \hat{c}_1|] = \text{sign}[(\partial a_2/\partial |\hat{c}_h - \hat{c}_1|)(-d_1"d_2"B\sigma_c^2n) > 0.$$

$$\text{sign}[dkB/d|\hat{c}_h - \hat{c}_1|] = \text{sign}[(\partial a_2/\partial |\hat{c}_h - \hat{c}_1|)(d_1"d_2"B\sigma_c^2(n+n_1)+Bc_1^2d_2") < 0.$$

$$\text{sign}[d(k+1)B/d|\hat{c}_h - \hat{c}_1|] = \text{sign}[(\partial a_2/\partial |\hat{c}_h - \hat{c}_1|)(d_1"d_2"B\sigma_c^2n_1+Bc_1^2d_2") < 0.$$

Q.E.D.

Remark. When the gap between cost targets is getting larger (i.e. smaller  $\hat{c}_h$  or larger  $\hat{c}_1$ ) to reach, the principal needs to provide more incentive for the production manager.

#### The Effect of The Difference of Attainable Quality Levels and The Marginal Probability of Getting High Quality Materials (h-1) and $P_h'$

Result 4.13. The procurement (production) manager's sharing ratio increases (decreases) with an increase of either the difference of attainable materials quality or the marginal probability of getting the getting high quality materials. The sharing pool, however, increases with both changes.

Proof. Result 4.13 follows by examining the following signs.

$$\text{sign}[dB/d(h-1)] = \text{sign}[(\partial a_2/\partial (h-1))(-d_1"d_2"B\sigma_c^2n) < 0.$$

$$\text{sign}[dkB/d(h-1)] = \text{sign}[(\partial a_2/\partial (h-1))(d_1"d_2"B\sigma_c^2(n+n_1)+Bc_1^2d_2") > 0.$$

$$\text{sign}[d(k+1)B/d(h-1)] = \text{sign}[(\partial a_2/\partial (h-1))(d_1"d_2"B\sigma_c^2n_1+Bc_1^2d_2") > 0.$$

$$\text{sign}[dB/dP_h'] = \text{sign}[-2d_1"B\sigma_c^2n_{xy}] < 0.$$

$$\text{sign}[dkB/dP_h'] = \text{sign}[2xyB(c_1^2+d_1"\sigma_c^2(n+n_1))] > 0.$$

$$\text{sign}[d(k+1)B/dP_h'] = \text{sign}[2xyB(c_1^2+d_1"\sigma_c^2n_1)] > 0.$$

Q.E.D.

Remark. When the attainable materials quality differs significantly, the procurement manager is encouraged to expend personal effort to get higher quality materials.

The Effect of The Degree of Absolute Effort Aversion ( $-d_1''/d_1'$  and  $-d_2''/d_2'$ )

Result 4.14. Each manager's sharing ratio decreases with an increase in his own degree of effort aversion, and increases when the other person becomes more effort averse. The sharing pool decreases when the managers are more effort averse.

Proof. Let  $v_1 = -d_1''/d_1'$  and  $v_2 = -d_2''/d_2'$ . Result 4.14 follows by examining the following signs.

$$\text{sign}[dB/dv_1] = \text{sign}[(\partial a_1/\partial v_1)Bd_1''(d_2''\sigma_c^2(n+n_2)+(xP_h')^2)] < 0.$$

$$\text{sign}[dkB/dv_1] = \text{sign}[-(\partial a_1/\partial v_1)Bd_1''d_2''\sigma_c^2n] > 0.$$

$$\text{sign}[d(k+1)B/dv_1] = \text{sign}[(\partial a_1/\partial v_1)Bd_1''(d_2''\sigma_c^2n_2+(xP_h')^2)] < 0.$$

$$\text{sign}[dB/dv_2] = \text{sign}[-(\partial a_2/\partial v_2)Bd_1''d_2''\sigma_c^2n] > 0.$$

$$\text{sign}[dkB/dv_2] = \text{sign}[(\partial a_2/\partial v_2)(Bd_1''d_2''\sigma_c^2(n+n_1)+Bc_1^2d_2'')] < 0.$$

$$\text{sign}[d(k+1)B/dv_2] = \text{sign}[(\partial a_2/\partial v_2)(Bd_1''d_2''\sigma_c^2n_1+Bc_1^2d_2'')] < 0.$$

Q.E.D.

Summary

In this chapter, a hierarchical agency model was proposed to discuss the interaction of procurement and production departments under linear profit-sharing and target-setting compensation plans. The production manager is expected to expend effort to reduce production costs. The procurement manager is expected to expend effort to increase the chance of getting high-quality materials which tends to reduce production costs. The principal would like to see the reduction of

production costs, provided that the higher price of better-quality materials does not offset the production-cost reduction.

Several interesting results have been discussed. Managers' compensations are tied to their positions when the principal lacks a costless monitoring mechanism, which explains a common practice in the real world. Comparing the second-best case with the first-best case, the principal has to provide a greater sharing pool in order to induce the managers' efforts, and each manager, in fact, works less hard. Agency costs, defined as the deviation from Pareto-optimal risk sharing from the principal's point of view, can be considered as the expected value of getting a perfect monitoring mechanism. In both first-best and second-best cases, profit-sharing compensation is always preferred to fixed salary from the principal's viewpoint. This result can explain why profit-sharing plans are getting more and more popular.

Detailed comparative statics were also discussed in this chapter. Results are quite robust and satisfy intuitive conjecture. When the attainable materials quality can be further improved, or the marginal chance of getting it is higher, the principal is willing to offer more incentives. The principal has to bear more risk once the managers are more effort averse or risk averse. The principal can also adjust the target costs to trigger proper effort levels. Diversification opportunities are always welcomed by the principal since the enhanced diversification forces the managers to bear a higher share of the volatile actual cost (i.e. a more negative  $\text{Cov}(R,C)$  corresponds to higher sharing ratios). The principal should always improve the

technology and methodology of production processes in order to reduce the processes' variances. Extensions and further research of this model are discussed in chapter 6.

## CHAPTER 5

### DEGREE OF SUPERVISION AND MORAL HAZARD IN HIERARCHICAL CONTROL

The study of hierarchical organizations was first developed by Simon (1957), Simon and Ijiri (1977) and Lydall (1968). They attempt to explain the observed skewness of the upper tail of the wages distribution. Simon and Lydall both assume that (1) the employees of a firm supervise employees at the immediate lower level, (2) the lowest-level employees are the only production workers, (3) the span of control, which is defined as the supervisor/supervisee ratio, is constant across layers, and (4) the wage at any layer is a constant multiple of the wage at the layer below. Studies by others use those assumptions, in part. Simon and Lydall show that these assumptions generate a Pareto wage distribution, but they do not give an economic explanation of internal wage differentials and labor utilization structure.

An endogenous explanation of the internal wage differentials is proposed by Calvo and Wellisz (1979). Employees differ in quality because of native ability, education, diligence, or other factors. Calvo and Wellisz show that (1) the employees' quality and wage increase with their hierarchical position, and (2) the imposition of a minimum wage for production workers increases the quality and quantity of production workers, and reduces the wage, quality and number of the supervisors. Calvo and Wellisz's results help to explain the

hierarchical differentials in worker quality, wages and degree of supervision. Production workers, however, are assumed to work at full efficiency at all levels. That is, there is no interest conflict between the supervisor and subordinates.

Stiglitz (1975) analyzed the role of incentives, risk, and information in determining the structure of a hierarchy. In Stiglitz' study, the wage is not a constant. The choice of a payment system depends on attitudes toward risk, effort-supply elasticities, the sources and magnitude of the uncertainties, and the nature of the supervision used in the employment relationship. Stiglitz concluded that it pays either not to monitor at all, or to monitor at a finite level. The costs of supervision are not considered in his models.

From an empirical study, Rosen (1982) indicates that firm size and earnings distributions follow similar functional forms and exhibit similar general appearances. Williamson (1967) investigates the size of firm theoretically using similar assumptions as in Simon and Lydall, and concludes that limited firm size is due to a loss of control factor which is given exogenously. William's conclusions have been challenged by Mirrlees (1976) and Calvo (1977), who argue that the height of the hierarchical pyramid does not set a limit to firm size unless ad hoc considerations are introduced. Calvo and Wellisz (1978) justify that hierarchical loss of control depends crucially on the nature of the supervision process. If the employees are aware of the times at which they are not being watched, loss of control may impose such a limit on firm size.

These previous studies consider labor to be the only factor of production, and assume that the hierarchy layers are established exogenously. But, a recent survey conducted by the Institute of Industrial Engineers indicates that 97 percent of the respondents believe that a closer relationship between employees and managers is an important factor for achieving increases in productivity. Significantly, 88 percent strongly agree that lack of communication and cooperation between departments reduces productivity (OR/MS Today, 1990). Supervisory input might therefore also be considered to be a factor of production. Supervision is more than just monitoring; it also serves as a communication tool and has certain direct effects on production.

Stiglitz (1975) mentions various reasons for using supervisors: supervisors as monitors of input, supervisors as monitors of ability, supervisors as monitors of output, and supervisors acting as a substitute for a payment system. It is the role of supervisors as monitors of labor input that is elaborated in this chapter. Supervision of the production process is the first level of management, involving direct interactions and communication with production workers. Thurley and Hamblin (1962) define the production supervisors as managers who spent over ten percent of their time on the shopfloor, and who were concerned with shopfloor control within different production systems. The presence of a supervisory system also implies that the monitoring of employees' efforts is less costly than the direct measurement of their marginal product. That is, although the final outputs are observable by



the principal, it is hardly possible to differentiate production input and supervisory input.

This chapter, unlike previous studies, focuses on the necessity of production supervision and on the payoff of establishing an expanded hierarchical structure. The employees have discretion to choose their effort levels, and the supervisory efforts have direct effects on production. There are five models discussed and compared in this chapter. It is shown that, in most cases, the principal that is involved in both production and other economic activities would like to pay for supervisors to directly manage the production process. The first model considers a two-layered hierarchy where the principal doubles as a supervisor and the workers expend full effort. The second model considers the same two-layered hierarchy except that the workers' moral hazard problem exists. The third model considers a three-layered structure where the principal mandates managers to supervise production and the moral hazard problem does not exist in the organization. The fourth model considers the same three-layered hierarchy except that a moral hazard problem exists among the workers. Finally, the fifth model considers the three-layered structure with the moral hazard problem at both the production and supervisory levels. In what follows, these models and their optimal solutions are discussed. Then comparisons of these models are analyzed. This chapter concludes with a summary of the findings.

### The Models

Consider the following situation. The single principal (owner) of an organization hires  $m_1$  homogeneous production workers. The principal can supervise the workers himself, or he can hire  $m_2$  homogeneous managers to supervise the production line and then he, in turn, monitors the managers. Output  $q$  is a function of effective labor and supervisory inputs. I further assume the production function to be Cobb-Douglas

$$q = k(m_1\alpha)^\epsilon z^\eta, \quad (5.1)$$

where  $k$  is a scale parameter that measures the average productivity,  $m_1\alpha$  stands for the effective labor force and  $\alpha \leq 1$ ,  $\epsilon$  and  $\eta$  are output elasticities with  $0 < \epsilon, \eta < 1$ , and  $z$  stands for the supervisor input. With this production function, the workers cannot produce profitable products without any input from their superiors. That is, the workers lack the knowledge of managing and directing production themselves. If the principal himself supervises the workers, then  $z$  is the principal's effort,  $\gamma$ , expended on production, i.e.  $z = \gamma$  and  $\gamma \leq 1$ . If the principal hires  $m_2$  managers to supervise  $m_1$  workers, then the supervisor inputs  $z = m_2\beta$ , where  $\beta$  is each manager's supervisory effort and  $\beta \leq 1$ . These homogeneous  $m_2$  managers cannot jointly monitor the workers, but must spend time with each worker to obtain useful outputs. Since the supervisory input has its maximum limit, it is assumed that the production function exhibits decreasing returns to scale, i.e.  $\epsilon + \eta < 1$  (Varian, 1984, p. 20). Therefore, by checking the Hessian matrix, the production function is strictly concave for all positive values of  $m_1\alpha$  and  $z$  (Henderson and Quandt, 1980, p. 72).

Monitoring, to be effective, requires the imposition of penalties for substandard work. Let  $P_1$  be the probability of a worker's performance being checked by his supervisor, which is called "the degree of supervision" by Calvo and Wellisz (1979). If a worker is not checked, he is presumed to have worked at maximum intensity (i.e.  $\alpha=1$ ) and he is paid a wage  $S$ . If a worker's performance is checked, his effort level,  $\alpha$ , is revealed and he is paid  $\alpha S$ , involving a penalty equal to  $(1-\alpha)S$ . Employees are unaware of the times at which they are being checked. This payment scheme represents a mixture of a flat-rate scheme (applicable to employee whose performance is not checked), and a piece-rate scheme (applicable to employees whose work is checked). To gain in clarity and simplicity mathematically, following Calvo and Wellisz (1978), it is assumed that the penalties do not accrue to the principal in the form of cost savings. The penalties, if any, are retained for employee-related events such as social activities.

The worker's Von Neumann-Morgenstern utility function is defined over his payment and his personal expenditure on effort; more concretely, it is assumed that

$$\begin{aligned} U(\alpha; S) &= [S(1-P_1) + \alpha S P_1] - (d_1 \alpha)^{\delta_1} \\ &= S[1 - (1-\alpha)P_1] - (d_1 \alpha)^{\delta_1} \geq 0, \end{aligned} \tag{5.2}$$

where  $d_1$  represents the disutility of effort. The worker's objective is to choose his effort level to maximize his utility function (5.2). It is assumed that the employee is risk neutral and effort averse. Thus  $\delta_1 > 1$ . Since  $U(\alpha; S)$  is concave in  $\alpha$ , the first-order condition is both

necessary and sufficient for optimality. Hence the worker's optimal effort level  $\alpha^*$  is equal to  $(SP_1/\delta_1)^{1/(\delta_1-1)}(1/d_1)$ .

Examining the above function, if the wage  $S$  is too low, the worker does not expend full effort even under perfect monitoring ( $P_1=1$ ). To avoid this case, let  $S=\delta_1 d_1^{(\delta_1-1)}$ . (5.3)

The worker expends full effort ( $\alpha_1=1$ ) under perfect monitoring. Thus the worker's optimal effort level depends crucially on the degree of supervision alone, i.e.  $\alpha^*=(P_1)^{1/(\delta_1-1)}$ . (5.4)

The worker's effort level increases with an increase of the degree of supervision. From (5.2) and (5.3),  $U=\delta_1 d_1^{(\delta_1-1)}[1-(1-\alpha)P_1-(d_1/\delta_1)\alpha^{\delta_1}]$ . Since the minimum utility guaranteed to the worker is zero from (5.2), it is assumed that  $\delta_1 \geq d_1 > 1$ . ( $\delta_1$  needs to be further restricted to be greater than two in the later part of this chapter.) This assumption implies that only a certain type of employees will be attracted to work.

Substituting (5.4) in (5.2), at the optimum the worker's utility function becomes

$$U^*(S, \alpha) = S[1 - P_1 + (1 - d_1/\delta_1)P_1^{\delta_1/(\delta_1-1)}].$$

Taking the first derivative of  $U^*$  with respect to  $P_1$  yields

$$\partial U^*/\partial P_1 = S[-1 + ((\delta_1 - d_1)/(\delta_1 - 1))P_1^{1/(\delta_1-1)}].$$

The worker's utility decreases with an increase of the degree of supervision, given that  $\delta_1 \geq d_1 > 1$ .

The degree of supervision is defined as a function of the number of effective supervisors per supervisee (Calvo and Wellisz, 1979). At the production workers' level,  $P_1 = h_1(\gamma/m_1) \leq 1$  if the principal conducts supervision;  $P_1 = h_1(\beta m_2/m_1) \leq 1$  if the managers are hired. For simplicity,

let  $h_1$  be linear and  $h_1'=1$ . Thus, the degree of supervision at the workers' level,  $P_1$ , equals  $\gamma/m_1$  or  $\beta m_2/m_1$ . (5.5)

Besides production, the principal is also involved in other economic activities, which provide him maximum return  $R$  if he spends all of his time (effort) on those activities. The profit-maximizing principal's questions are: How much supervisory effort should he expend? How many production workers should be hired? Should the managers be hired, resulting in one more layer in the organization, to release him from direct supervision of production? The following models are proposed in an attempt to answer the above questions.

#### Model 5.1: Principal as Supervisor; No Moral Hazard

The first model considers the case where the principal himself supervises the production workers and the workers are willing to expend full effort regardless of the degree of supervision, i.e.  $\alpha=1$ . Let the unit product net price be normalized to one. That is, the firm faces a perfectly competitive market for its final products. The principal seeks to

$$\begin{aligned} \text{MAX}_{m_1, \gamma} \quad & \pi = q - m_1 S + R(1-\gamma) \\ & = k(m_1 \alpha)^\epsilon \gamma^\eta - m_1 S + R(1-\gamma) \\ \text{Subject to} \quad & \end{aligned} \quad (5.6)$$

$$\alpha=1, S=\delta_1 d_1^{\delta_1-1}$$

$$0 \leq \gamma \leq 1, m_1 > 0.$$

Profit  $\pi$  is strictly concave by the assumption that  $\epsilon+\eta < 1$ . The first-order conditions of (5.6) with respect to  $m_1$  and  $\gamma$  are:

$$\epsilon q/m_1 = S, \text{ and} \quad (5.7)$$

$$\eta q/\gamma = R. \quad (5.8)$$

Comparing (5.7) and (5.8), and from the definition of the optimal degree of supervision (5.5),

$$P_1^I = \gamma/m_1 = S\eta/\epsilon R < 1. \quad (5.9)$$

(Hereafter, the notations I, II, III, IV, V are not superscripts.

Rather, they indicate the order of the models.)

Following Ouchi and Dowling's (1974) definition, the span of control,  $N$ , is defined as the ratio of the number of subordinates to the number of immediate superiors. Substituting (5.9) in (5.7), the optimal span of control of the principal (which is also the optimal employment of production workers) is

$$N_1^I = m_1^I = \gamma^I \epsilon R / S \eta = [k(\epsilon/S)^{1-\eta}(\eta/R)^\eta]^{1/(1-\epsilon-\eta)}, \quad (5.10)$$

and the principal's optimal supervisory effort is

$$\gamma^I = m_1^I S \eta / \epsilon R = [k(\epsilon/S)^\epsilon(\eta/R)^{1-\epsilon}]^{1/(1-\epsilon-\eta)}. \quad (5.11)$$

From (5.10) and (5.11),  $m_1^I$  and  $\gamma^I$  increase with an increase of  $k$  and a decrease of  $S$  or  $R$ .

The principal's maximum profit level is

$$\begin{aligned} \pi^I &= (m_1^I S / \epsilon)(1 - \epsilon - \eta) + R \\ &= [k(\epsilon/S)^\epsilon(\eta/R)^\eta]^{1/(1-\epsilon-\eta)} + R. \end{aligned} \quad (5.12)$$

Since  $\epsilon + \eta < 1$ , the principal is better off keeping the production line running. The overall profits  $\pi^I$  consist of the profits generated from production

$$\pi_p^I = (m_1^I S / \epsilon)(1 - \epsilon) = (1 - \epsilon)[k(\epsilon/S)^\epsilon(\eta/R)^\eta]^{1/(1-\epsilon-\eta)}, \quad (5.13)$$

and the profits generated from other economic activities,

$$\pi_o^I = R(1 - \gamma^I). \quad (5.14)$$

Even if the workers are at full efficiency, the principal still has to expend some effort to manage and direct them toward a profitable direction. This supervisory responsibility incurs some opportunity costs for the principal:

$$L^I = R\gamma^I. \quad (5.15)$$

Each worker's utility is at

$$U^I(S, \alpha) = S - d_1 \delta_1. \quad (5.16)$$

#### Model 5.II: Principal as Supervisor; Workers with Moral Hazard

Suppose the workers are utility maximizers. Then the principal seeks to

$$\begin{aligned} \text{MAX}_{m_1, \gamma} \quad & \pi = q - m_1 S + R(1-\gamma) \\ & = k(m_1 \alpha)^\epsilon \gamma^\eta - m_1 S + R(1-\gamma) \end{aligned}$$

Subject to

$$\alpha = (\gamma/m_1)^{1/(\delta_1-1)}, \quad S = \delta_1 d_1 \delta_1^{-1}$$

$$0 \leq \gamma \leq 1, \quad m_1 > 0.$$

Substituting the constraints into the objective function yields

$$\pi = k m_1^{\epsilon(\delta_1-2)/(\delta_1-1)} \gamma^{\eta+\epsilon/(\delta_1-1)} - m_1 S + R(1-\gamma). \quad (5.17)$$

The profit function is strictly concave provided  $\delta_1 > 2$  and  $\epsilon + \eta < 1$ . The first-order conditions of (5.17) with respect to  $m_1$  and  $\gamma$  are:

$$(\epsilon q/m_1)(\delta_1-2)/(\delta_1-1) = S, \quad \text{and} \quad (5.18)$$

$$\eta q/\gamma + \epsilon q/\gamma(\delta_1-1) = R. \quad (5.19)$$

Comparing (5.18) and (5.19), the optimal degree of supervision is

$$P_1^{II} = \gamma/m_1 = S(\eta(\delta_1-1)+\epsilon)/(\delta_1-2)R\epsilon < 1. \quad (5.20)$$

Therefore, the optimal production effort is

$$\alpha^{II} = (\gamma/m_1)^{1/(\delta_1-1)} = [S(\eta(\delta_1-1)+\epsilon)/(\delta_1-2)R\epsilon]^{1/(\delta_1-1)} < 1. \quad (5.21)$$

Substituting (5.20) in (5.18), the optimal span of control of the Principal and the optimal employment of production workers are

$$\begin{aligned} N_1^{II} &= m_1^{II} = \gamma R \epsilon (\delta_1 - 2) / (\eta (\delta_1 - 1) + \epsilon) S \\ &= \{ (k(\delta_1 - 2) / (\delta_1 - 1)) (\epsilon / S)^{1 - (\eta + \epsilon / (\delta_1 - 1))} [(\eta (\delta_1 - 1) + \epsilon) / (\delta_1 - 2) R]^{\eta + \epsilon / (\delta_1 - 1)} \}^{1 / (1 - \epsilon - \eta)} \\ &> 0, \end{aligned} \quad (5.22)$$

and the optimal supervisory effort is

$$\begin{aligned} \gamma^{II} &= m_1^{II} S (\eta (\delta_1 - 1) + \epsilon) / (\delta_1 - 2) R \epsilon \\ &= \{ (k(\delta_1 - 2) / (\delta_1 - 1)) (\epsilon / S)^{\epsilon (\delta_1 - 2) / (\delta_1 - 1)} [(\eta (\delta_1 - 1) + \epsilon) / (\delta_1 - 2) R]^{1 - \epsilon (\delta_1 - 2) / (\delta_1 - 1)} \}^{1 / (1 - \epsilon - \eta)} \\ &< 1. \end{aligned} \quad (5.23)$$

From (5.17), (5.20) and (5.22), the principal's maximum profit is

$$\begin{aligned} \pi^{II} &= (m_1^{II} S / \epsilon) (1 - \epsilon - \eta) (\delta_1 - 1) / (\delta_1 - 2) + R = (\gamma^{II} R (\delta_1 - 1) / (\eta (\delta_1 - 1) + \epsilon)) (1 - \epsilon - \eta) + R \\ &= (1 - \epsilon - \eta) \{ (k(\delta_1 - 2) / (\delta_1 - 1))^{\epsilon + \eta} (\epsilon / S)^{\epsilon (\delta_1 - 2) / (\delta_1 - 1)} \\ &\quad [(\eta (\delta_1 - 1) + \epsilon) / (\delta_1 - 2) R]^{\eta + \epsilon / (\delta_1 - 1)} \}^{1 / (1 - \epsilon - \eta)} + R. \end{aligned} \quad (5.24)$$

The overall profits  $\pi^{II}$  consist of the profit generated from production,

$$\begin{aligned} \pi_p^{II} &= (m_1^{II} S / \epsilon) ((\delta_1 - 1) / (\delta_1 - 2) - \epsilon) \\ &= ((\delta_1 - 1) / (\delta_1 - 2) - \epsilon) [k(\epsilon / S)^{\epsilon (\delta_1 - 2) / (\delta_1 - 1)} \\ &\quad [(\eta (\delta_1 - 1) + \epsilon) / (\delta_1 - 2) R]^{\eta + \epsilon / (\delta_1 - 1)} \}^{1 / (1 - \epsilon - \eta)}, \end{aligned} \quad (5.25)$$

and the profits generated from other economic activities,

$$\pi_o^{II} = R(1 - \gamma^{II}) = \pi^{II} - \pi_p^{II}. \quad (5.26)$$

Each worker's optimal utility is

$$U^{II}(S, \alpha) = S [1 - P_1^{II} + (1 - d_1 / \delta_1) (P_1^{II})^{\delta_1 / (\delta_1 - 1)}]. \quad (5.27)$$

Comparing model 5.I with model 5.II (details are discussed in Result 5.1.), the principal's overall profits decrease due to his limited degree of supervision and the moral hazard problem. The decreased profits suggest the rationale for the existence of another



layer (i.e. supervising managers) to release the principal from supervising the production workers directly.

Consider the case that  $m_2$  managers are hired to supervise production, and the principal monitors the managers' performance instead. The manager's contract is similar to the worker's. Let  $P_2$  be the probability of a manager's performance being checked by the principal. If the manager's performance is not checked, he is presumed to have worked at maximum intensity (i.e.  $\beta=1$ ) and is paid a salary  $M$ . If the manager's performance is checked, his level of effort  $\beta$  is revealed and he is paid  $\beta M$ , involving a penalty equal to  $(1-\beta)M$ . Hereafter, it is assumed that the return from investment in other economic activities is greater than the manager's and the worker's payment, and the manager's salary is higher than the worker's wages, i.e.  $R > M > S$ . The manager's Von Neumann-Morgenstern utility is defined over his payment and his personal expenditure on effort; more concretely, it is assumed that

$$\begin{aligned} V(M, \beta) &= [M(1-P_2) + \beta MP_2] - (d_2 \beta)^{\delta_2} \\ &= M[1 - (1-\beta)P_2] - (d_2 \beta)^{\delta_2} \geq 0, \end{aligned} \quad (5.28)$$

where  $d_2$  is the manager's disutility of effort. From the first-order condition, the manager's optimal supervising effort is

$$\beta^* = [MP_2/\delta_2]^{1/(\delta_2-1)} (1/d_2). \quad (5.29)$$

Assuming the manager expends full effort ( $\beta=1$ ) under perfect monitoring ( $P_2=1$ ), his payment  $M$  should equal  $\delta_2 d_2^{\delta_2-1}$ . (5.30)

Thus  $\beta^* = P_2^{1/(\delta_2-1)}$ . (5.31)

Following the same points as in the worker's case, it is assumed that  $\delta_2 \geq d_2 > 1$ . The degree of supervision at this second layer,  $P_2$ , is defined as  $h_2(\gamma/m_2) \leq 1$ . For simplicity, let  $h_2$  be linear and  $h_2' = 1$ . Thus,  $P_2 = \gamma/m_2$ . Three models are considered at this three-level hierarchy as follows.

#### Model 5.III: Manager as Supervisor; No Moral Hazard Exists

Suppose the moral hazard problem does not exist in this three-layered organization. The principal seeks to

$$\begin{aligned}
 \text{MAX}_{m_1, m_2, \gamma} \quad & \pi = q - m_1 S - m_2 M + R(1-\gamma) \\
 & = k(m_1 \alpha)^\epsilon (m_2 \beta)^\eta - m_1 S - m_2 M + R(1-\gamma) \\
 \text{Subject to} \quad & \alpha = 1, \quad S = \delta_1 d_1^{\delta_1 - 1} \\
 & \beta = 1, \quad M = \delta_2 d_2^{\delta_2 - 1} \\
 & 0 \leq \gamma \leq 1, \quad m_1 > 0, \quad m_2 > 0.
 \end{aligned} \tag{5.32}$$

Substituting the constraints in the objective function yields

$$\pi = k m_1^\epsilon m_2^\eta - m_1 S - m_2 M + R(1-\gamma). \tag{5.33}$$

This profit function is strictly concave given that  $\epsilon + \eta < 1$ . It is straight forward to see that the principal's optimal supervisory effort is at the minimum, i.e.  $\gamma^{III} = 0$ . The first-order conditions of (5.33) with respect to  $m_1$  and  $m_2$  are

$$\epsilon q / m_1 = S, \text{ and} \tag{5.34}$$

$$\eta q / m_2 = M. \tag{5.35}$$

Comparing (5.34) and (5.35), the optimal span of control of the supervisory manager is

$$N_1^{III} = m_1 / m_2 = \epsilon M / S \eta. \tag{5.36}$$

Substituting (5.36) in (5.34), the optimal employment of production workers is

$$m_1^{III} = [k(\epsilon/S)^{1-\eta}(\eta/M)^\eta]^{1/(1-\epsilon-\eta)}, \quad (5.37)$$

and the optimal number of managers (which is also the span of control of the principal) is

$$N_2^{III} = m_2^{III} = [k(\epsilon/S)^\epsilon(\eta/M)^{1-\epsilon}]^{1/(1-\epsilon-\eta)}. \quad (5.38)$$

From (5.37) and (5.38),  $m_1^{III}$  and  $m_2^{III}$  increase with an increase of  $k$  and a decrease of  $S$  or  $M$ .

From (5.33), (5.36) and (5.37), the principal's maximum profit

$$\begin{aligned} \pi^{III} &= (m_1^{III}S/\epsilon)(1-\epsilon-\eta)+R \\ &= (1-\epsilon-\eta)[k(\epsilon/S)^\epsilon(\eta/M)^\eta]^{1/(1-\epsilon-\eta)} + R. \end{aligned} \quad (5.39)$$

The overall profits  $\pi^{III}$  consist of the profits generated from production

$$\pi_p^{III} = (m_1^{III}S/\epsilon)(1-\epsilon-\eta), \quad (5.40)$$

and the profits generated from other economic activities

$$\pi_o^{III} = R. \quad (5.41)$$

The principal does not have an opportunity loss of running production, i.e.

$$L^{III} = 0. \quad (5.42)$$

Each worker's optimal utility is

$$U^{III} = S-d_1^{\delta_1}, \quad (5.43)$$

and each manager's optimal utility is

$$V^{III} = M-d_2^{\delta_2}. \quad (5.44)$$

Model 5.IV: Manager as Supervisor with Full Efficiency

At this three-layered hierarchy, suppose only the manager is at full efficiency regardless of the degree of supervision and the workers are utility maximizers. Then the principal seeks to

$$\begin{aligned}
 \text{MAX}_{m_1, m_2, \gamma} \quad & \pi = q - m_1 S - m_2 M + R(1-\gamma) \\
 & = k(m_1 \alpha)^\epsilon (m_2 \beta)^\eta - m_1 S - m_2 M + R(1-\gamma) \\
 \text{Subject to} \quad & \alpha = (m_2 \beta / m_1)^{1/(\delta_1 - 1)} < 1, \quad S = \delta_1 d_1^{\delta_1 - 1} \\
 & \beta = 1, \quad M = \delta_2 d_2^{\delta_2 - 1} \\
 & 0 \leq \gamma \leq 1, \quad m_1 > 0, \quad m_2 > 0.
 \end{aligned} \tag{5.45}$$

Substituting the constraints into the objective function yields

$$\pi = k m_1^{\epsilon(\delta_1 - 2)/(\delta_1 - 1)} m_2^{\eta + \epsilon/(\delta_1 - 1)} - m_1 S - m_2 M + R(1-\gamma). \tag{5.46}$$

This profit function is strictly concave if  $\delta_1 > 2$ ,  $\delta_2 > 2$  and  $\epsilon + \eta < 1$ . It is straight forward to see that the optimal effort from the principal is at the minimum level, i.e.  $\gamma = 0$ . The first-order conditions of (5.46) with respect to  $m_1$  and  $m_2$  are

$$(\epsilon q / m_1)(\delta_1 - 2)/(\delta_1 - 1) = S, \text{ and} \tag{5.47}$$

$$(q / m_2)(\eta + \epsilon/(\delta_1 - 1)) = M. \tag{5.48}$$

Comparing (5.47) and (5.48), the optimal degree of supervision at the first level is

$$P_1^{IV} = \beta m_2 / m_1 = m_2 / m_1 = S(\eta(\delta_1 - 1) + \epsilon)/(\delta_1 - 2)M\epsilon < 1. \tag{5.49}$$

Therefore the manager's optimal span of control is

$$N_1^{IV} = m_1 / m_2 = M\epsilon(\delta_1 - 2)/(\eta(\delta_1 - 1) + \epsilon)S > 1, \tag{5.50}$$

and the worker's optimal production effort is

$$\alpha^{IV} = (P_1^{IV})^{1/(\delta_1 - 1)} = [S(\eta(\delta_1 - 1) + \epsilon)/(\delta_1 - 2)M\epsilon]^{1/(\delta_1 - 1)}. \tag{5.51}$$

Substituting (5.49) in (5.47), the optimal employment of production workers is

$$m_1^{IV} = \{ (k(\delta_1-2)/(\delta_1-1))(\epsilon/S)^{1-(\eta+\epsilon/(\delta_1-1))} \\ [(\eta(\delta_1-1)+\epsilon)/(\delta_1-2)M]^{\eta+\epsilon/(\delta_1-1)}\}^{1/(1-\epsilon-\eta)}, \quad (5.52)$$

and the optimal number of managers, i.e the principal's span of control, is

$$N_2^{IV} = m_2^{IV} \\ = \{ (k(\delta_1-2)/(\delta_1-1))(\epsilon/S)^{\epsilon(\delta_1-2)/(\delta_1-1)} \\ [(\eta(\delta_1-1)+\epsilon)/(\delta_1-2)M]^{1-\epsilon(\delta_1-2)/(\delta_1-1)}\}^{1/(1-\epsilon-\eta)}. \quad (5.53)$$

From (5.46), (5.49) and (5.52), the principal's maximum profit is

$$\pi^{IV} = (m_1^{IV}S/\epsilon)(1-\epsilon-\eta)(\delta_1-1)/(\delta_1-2)+R. \\ = (1-\epsilon-\eta)\{k((\delta_1-2)/\delta_1-1))^{\epsilon+\eta}(\epsilon/S)^{\epsilon(\delta_1-2)/(\delta_1-1)} \\ [(\eta(\delta_1-1)+\epsilon)/(\delta_1-2)M]^{\eta+\epsilon/(\delta_1-1)}\}^{1/(1-\epsilon-\eta)}. \quad (5.54)$$

The overall profits  $\pi^{IV}$  consist of the profits generated from production

$$\pi_p^{IV} = (m_1^{IV}S/\epsilon)((\delta_1-1)/(\delta_1-2)-\epsilon), \quad (5.55)$$

and the profits generated from other economic activities

$$\pi_o^{IV} = R. \quad (5.56)$$

The principal does not have an opportunity loss of running production, i.e.

$$L^{IV} = 0. \quad (5.57)$$

Each worker's optimal utility is

$$U^{IV} = S[1-P_1^{IV}+(1-d_1/\delta_1)(P_1^{IV})^{\delta_1/(\delta_1-1)}], \quad (5.58)$$

and each manager's optimal utility is

$$V^{IV} = M-d_2^{\delta_2}. \quad (5.59)$$

Model 5.V: Manager as Supervisor with Moral Hazard

Suppose both workers and managers are utility maximizers in this three-level hierarchy. The principal seeks to

$$\begin{aligned}
 \text{MAX}_{m_1, m_2, \gamma} \quad & \pi = q - m_1 S - m_2 M + R(1-\gamma) \\
 & = k(m_1 \alpha)^\epsilon (m_2 \beta)^\eta - m_1 S - m_2 M + R(1-\gamma) \\
 \text{Subject to} \quad & \alpha = (m_2 \beta / m_1)^{1/(\delta_1 - 1)} < 1, \quad S = \delta_1 d_1^{\delta_1 - 1} \\
 & \beta = (\gamma / m_2)^{1/(\delta_2 - 1)} < 1, \quad M = \delta_2 d_2^{\delta_2 - 1} \\
 & 0 \leq \gamma \leq 1, \quad m_1 > 0, \quad m_2 > 0.
 \end{aligned} \tag{5.60}$$

Substituting the constraints in the objective function yields

$$\pi = k m_1^{\epsilon(\delta_1 - 2)/(\delta_1 - 1)} m_2^{(\eta + \epsilon/(\delta_1 - 1))(\delta_2 - 2)/(\delta_2 - 1)} \gamma^{(\eta + \epsilon/(\delta_1 - 1))/(\delta_2 - 1)} - m_1 S - m_2 M + R(1-\gamma). \tag{5.61}$$

Checking the Hessian matrix, (5.61) is strictly concave given  $\delta_1 > 2$ ,  $\delta_2 > 2$  and  $\epsilon + \eta < 1$ . Suppose the interior solution exists. The first-order conditions of (5.61) with respect to  $m_1$ ,  $m_2$  and  $\gamma$  are

$$(\epsilon q / m_1) (\delta_1 - 2) / (\delta_1 - 1) = S, \text{ and} \tag{5.62}$$

$$(q / m_2) (\eta + \epsilon / (\delta_1 - 1)) (\delta_2 - 2) / (\delta_2 - 1) = M, \text{ and} \tag{5.63}$$

$$(q / \gamma) (\eta + \epsilon / (\delta_1 - 1)) / (\delta_2 - 1) = R. \tag{5.64}$$

Comparing (5.62) and (5.63), the manager's optimal span of control is

$$N_1^V = m_1 / m_2 = [M \epsilon (\delta_1 - 2) / (\eta (\delta_1 - 1) + \epsilon) S] (\delta_2 - 1) / (\delta_2 - 2). \tag{5.65}$$

Comparing (5.63) and (5.64), the optimal degree of supervision at the second layer is

$$P_2^V = \gamma / m_2 = M / R (\delta_2 - 2) < 1, \tag{5.66}$$

and the manager's optimal supervisory effort is

$$\beta^V = (\gamma / m_2)^{1/(\delta_2 - 1)} [M / R (\delta_2 - 2)]^{1/(\delta_2 - 1)} < 1. \tag{5.67}$$

From (5.65) and (5.66), the optimal degree of supervision for the first-layer workers is

$$P_1^V = \beta m_2 / m_1 = (m_2 / m_1) (\gamma / m_2)^{1/(\delta_2-1)} \\ = [(S(\eta(\delta_1-1)+\epsilon)/(\delta_1-2)M\epsilon)(\delta_2-2)/(\delta_2-1)](M/R(\delta_2-2))^{1/(\delta_2-1)} < 1, \quad (5.68)$$

and the optimal production effort is

$$\alpha^V = (P_1^V)^{1/(\delta_1-1)} \\ = \{[(S(\eta(\delta_1-1)+\epsilon)/(\delta_1-2)M\epsilon)(\delta_2-2)/(\delta_2-1)](M/R(\delta_2-2))^{1/(\delta_2-1)}\}^{1/(\delta_1-1)}. \quad (5.69)$$

Substituting (5.65), (5.66) in (5.61), the optimal employment of production workers is

$$m_1^V = \{(k(\delta_1-2)/(\delta_1-1))(\epsilon/S)^{1-(\eta+\epsilon/(\delta_1-1))} \\ [((\eta(\delta_1-1)+\epsilon)/(\delta_1-2)M)(\delta_2-2)/(\delta_2-1)]^{\eta+\epsilon/(\delta_1-1)} \\ (M/R(\delta_2-2))^{(\eta+\epsilon/(\delta_1-1))/(\delta_2-1)}\}^{1/(1-\epsilon-\eta)}, \quad (5.70)$$

and the optimal number of managers, i.e. the principal's span of control, is

$$N_2^V = m_2^V = \{(k(\delta_1-2)/(\delta_1-1))(\epsilon/S)^{\epsilon(\delta_1-2)/(\delta_1-1)} \\ [((\eta(\delta_1-1)+\epsilon)/(\delta_1-2)M)(\delta_2-2)/(\delta_2-1)]^{1-\epsilon(\delta_1-2)/(\delta_1-1)} \\ (M/R(\delta_2-2))^{(\eta+\epsilon/(\delta_1-1))/(\delta_2-1)}\}^{1/(1-\epsilon-\eta)}. \quad (5.71)$$

From (5.65), (5.66) and (5.70), the principal's supervising effort is

$$\gamma^V = m_2 M / R(\delta_2-2) = (m_1 S / R\epsilon(\delta_1-2))((\eta(\delta_1-1)+\epsilon)/(\delta_2-1)) \\ = \{(k(\delta_1-2)/(\delta_1-1))(\epsilon/S)^{\epsilon(\delta_1-2)/(\delta_1-1)} \\ [((\eta(\delta_1-1)+\epsilon)/(\delta_1-2)M)(\delta_2-2)/(\delta_2-1)]^{1-\epsilon(\delta_1-2)/(\delta_1-1)} \\ (M/R(\delta_2-2))^{(1-\epsilon-\eta)+(\eta+\epsilon/(\delta_1-1))/(\delta_2-1)}\}^{1/(1-\epsilon-\eta)} < 1. \quad (5.72)$$

From (5.61), (5.65), (5.66) and (5.70), the principal's maximum profit is

$$\pi^V = (m_1^V S / \epsilon)(1-\epsilon-\eta)(\delta_1-1)/(\delta_1-2) + R. \quad (5.73)$$

The overall profit  $\pi^V$  consist of the profits generated from production is

$$\pi_p^V = (m_1^V S / \epsilon) [ (1 - \epsilon - \eta)(\delta_1 - 1) / (\delta_1 - 2) + (\eta(\delta_1 - 1) + \epsilon) / (\delta_2 - 1) ], \quad (5.74)$$

and the profit generated from other economic activities is

$$\pi_o^V = R(1 - \gamma^V). \quad (5.75)$$

The principal's opportunity loss of running production is

$$L^V = R\gamma^V. \quad (5.76)$$

Each worker's optimal utility is

$$U^V = S[1 - P_1^{IV} + (1 - d_1 / \delta_1)(P_1^V)^{\delta_1 / (\delta_1 - 1)}], \quad (5.77)$$

and each manager's optimal utility is

$$V^V = M[1 - P_2^V + (1 - d_2 / \delta_2)(P_2^V)^{\delta_2 / (\delta_2 - 1)}]. \quad (5.78)$$

### The Comparison of the Models

This chapter is concerned with the necessity of production supervision and the payoff of establishing an expanded hierarchical structure. The desirability of the hierarchy is examined by comparing the above five models. Results are discussed as follows.

Result 5.1. Comparing model 5.I and Model 5.II, when the principal doubles as supervisor and the workers' moral hazard problems exist, the principal expends less effort on supervising production and fewer workers are employed. Although the degree of supervision is increased, overall profits and profits from production decrease.

Proof. Consider Model 5.I and Model 5.II. Comparing (5.9) and (5.20), the degree of supervision,

$$P_1^{II} = (\gamma / m_1)^{II} > P_1^I = (\gamma / m_1)^I > 0. \quad (5.79)$$



Considering (5.11) and (5.23) yields the ratio

$$\gamma^{II}/\gamma^I = \{((\delta_1-2)/(\delta_1-1))(S/\epsilon)^{\epsilon/(\delta_1-1)}(R/\eta)^{1-\epsilon} \\ [(\eta(\delta_1-1)+\epsilon)/(\delta_1-2)R]^{1-\epsilon(\delta_1-2)/(\delta_1-1)}\}^{1/(1-\epsilon-\eta)}. \quad (5.80)$$

Suppose  $\gamma^{II} \geq \gamma^I$ , then

$$(S/\epsilon)^{\epsilon/(\delta_1-1)}(R/\eta)^{1-\epsilon} \geq ((\delta_1-1)/(\delta_1-2))[R(\delta_1-2)/(\eta(\delta_1-1)+\epsilon)]^{1-\epsilon(\delta_1-2)/(\delta_1-1)} \\ > [R(\delta_1-2)/(\eta(\delta_1-1)+\epsilon)]^{1-\epsilon(\delta_1-2)/(\delta_1-1)}.$$

From (5.9),  $1 < S/\epsilon < R/\eta$ .

$$\text{Therefore, } (R/\eta)^{\epsilon/(\delta_1-1)}(R/\eta)^{1-\epsilon} > (S/\epsilon)^{\epsilon/(\delta_1-1)}(R/\eta)^{1-\epsilon} \\ > [R(\delta_1-2)/(\eta(\delta_1-1)+\epsilon)]^{1-\epsilon(\delta_1-2)/(\delta_1-1)}.$$

That is,  $R/\eta > R(\delta_1-2)/(\eta(\delta_1-1)+\epsilon)$ , which implies that  $\epsilon+\eta > 1$ . This is a contradiction to the assumption of decreasing return to scale.

$$\text{Hence } \gamma^{II} < \gamma^I. \quad (5.81)$$

$$\text{From (5.79) and (5.81), } m_1^{II} < m_1^I. \quad (5.82)$$

Comparing (5.12) and (5.24),

$$(\pi^{II}-R)/(\pi^I-R) = (\gamma^{II}/\gamma^I)(1+\epsilon/(\eta(\delta_1-1))) > 1. \quad \text{Hence } \pi^{II} < \pi^I. \quad (5.83)$$

$$\text{Comparing (5.14) and (5.26), } \pi_o^I/\pi_o^{II} = (1-\gamma^I)/(1-\gamma^{II}) < 1. \quad (5.84)$$

$$\text{Since } \pi = \pi_o + \pi_p, \text{ examining (5.83) and (5.84), } \pi_p^I > \pi_p^{II}. \quad \text{Q.E.D.}$$

Remark. Result 5.1. indicates that loss in production is due to the moral hazard problem and the principal's limited span of control. It then suggests the desirability of the principal to hire managers.

Result 5.2. If employees at all levels expend full effort regardless of monitoring schemes, it is to the principal's benefit to expand the hierarchy levels and organization size.

Proof. Compare Model 5.I. and Model 5.III. From (5.10) and (5.37),

$$(m_1^{III}/m_1^I)^{1-\epsilon-\eta} = (R/M)^\eta > 1. \quad \text{Therefore } m_1^{III} > m_1^I. \quad \text{From (5.12) and (5.39),}$$

$(\pi^{III}-R)/(\pi^I-R) = (m_1^{III}/m_1^I) > 1$ . The overall profit increases by adding one managerial level and hiring more production workers. Q.E.D.

Result 5.3. When moral hazard exists among production workers, it is to the principal's benefit to hire managers to supervise the workers in behalf of the principal, and more workers should be hired.

Proof. Compare Model 5.II and Model 5.IV. From (5.20) and (5.49), the degree of supervision at the first level is higher with managers as supervisors, and thus the workers expend more effort. From (5.22) and (5.52),  $(m_1^{IV}/m_1^{II})^{1-\epsilon-\eta} = (R/M)^{\eta+\epsilon/(\delta_1-1)} > 1$ . Hence more production workers are hired with managers as supervisors. From (5.24) and (5.54),  $(\pi^{IV}-R)/(\pi^{II}-R) = (m_1^{IV}/m_1^{II}) > 1$ . Comparing (5.25) with (5.55), (5.26) with (5.56), profits from both production and other economic activities are also increased when managers are hired. Q.E.D.

Result 5.4. When employees at all levels do not expend full effort, it is still to the principal's benefit to hire supervising managers if the workers are more effort averse and the managers are much less effort averse.

Proof. Compare Model 5.II and Model 5.V. From (5.20), (5.65), (5.66) and (5.68),

$$P_1^V = (m_2/m_1)(\gamma/m_2)^{1/(\delta_2-1)} > (m_2/m_1)(\gamma/m_2) = \gamma/m_1 = S(\eta(\delta_1-1)+\epsilon)/(\delta_1-2)(\delta_2-1)R\epsilon = P_1^{II}. \quad (5.85)$$

The degree of supervision at the first level is higher when managers are hired. Hence the workers expend more effort in Model 5.V. From (5.22) and (5.70),

Therefore  $m_1^V > m_1^{II}$  if (5.85) is greater than unity. Examining (5.85), the first term is less than unity. The second term approaches unity

when the workers are more effort averse (i.e. larger  $\delta_1$ ) and the managers are less effort averse (i.e. smaller  $\delta_2$ ). When  $m_1^V > m_1^{II}$ , comparing (5.24) with (5.73), (5.23) with (5.72), (5.26) with (5.75) yields  $\pi^V > \pi^{II}$ ,  $\gamma^V > \gamma^{II}$ ,  $\pi_p^V > \pi_p^{II}$ , and  $\pi_o^V < \pi_o^{II}$ . The principal expends more supervising efforts in the three-layered structure and his overall profits increase. Q.E.D.

### Summary

In this chapter, agency models are proposed to explain the necessity of production supervision and the payoff to establishing an expanded hierarchical structure. The following situation is considered: The single owner (principal) of an organization hires production workers to run production. The workers are effort averse and demand instruction and guidance for their jobs. The principal has a limited span of control and is involved in other economic activities. Instead of supervising production workers directly, it is shown that the principal is better off hiring managers as supervisors in many cases even under the existence of the moral hazard problem at all levels. Five models are discussed and compared. The first two models deal with a two-layered hierarchy, and the remaining models consider the three-level hierarchy. From the principal's point of view,  $\pi^{III} > \pi^I > \pi^{IV} > \pi^V > \pi^{II}$ . Results can be extended to higher-level hierarchies. Extensions and further research are discussed in Chapter 6.

## CHAPTER 6 CONCLUSION AND FURTHER RESEARCH

In this dissertation, hierarchical agency models are proposed to discuss agency relationships within different organizational structures under production environments. These models are established according to the expertise required in the hierarchy. There are three functional areas discussed in this study: quality assurance, procurement management, and production supervision. In what follows, a brief summary is given, and prospective research extensions of each chapter are discussed.

### Quality Assurance and Job Enlargement in Production Management

The integration of quality assurance and the production process is the major concern in this chapter. The quality control manager is responsible for establishing quality policies and monitoring the production process. The worker is responsible for both production and implementation of the quality policies. The worker is better off with his dual responsibilities, not only through increased compensation, but also through enlarged job interests. In general, both agents' contracts are increasing functions of the quality and quantity produced. Since the quality and quantity are nonpositively correlated, the quality assurance manager also shares some production risk. Detailed comparative statics were derived for an additive quality-enhancing technology and power utility functions. It is shown that the reductions

in the uncertainty and the unit production cost, and the increases in the quality mark-up price and the job-enlargement factors are encouraged by the principal. When alternative job opportunities become more attractive, the principal should modify the compensation plans with an increased salary and lowered commission rates.

This model assumes that the principal can observe the ex post quality and quantity. The ex post quality is a joint outcome of the manager's policy-setting effort and the worker's implementing effort. If a low quality level is observed after production, the principal cannot tell whether problems come from the manager or the worker. One possible extension is to assume that the principal gets a signal about the manager's policy-setting effort or the worker's implementing effort. Thus the compensation plans need to be designed to compensate each agent's fulfillment of his own responsibility. Another extension would be to assume that the principal is unable to observe the ex post quality standard. It is the quality assurance manager's responsibility to submit a report after production. Thus contracts need to be designed so as to induce truth-telling behavior.

Profit Sharing and Target Setting in Procurement Management The interaction of procurement and production managers under linear profit-sharing and target-setting compensation plans is the major concern of this chapter. The production manager is given the responsibility for reducing production costs. The procurement manager is responsible for purchasing quality materials. The quality of materials is used to adjust the cost target for the production department. Quality materials, on the one hand, reduce production costs due to less rework

or scratch; on the other hand, they reduce the principal's capital for investment due to higher purchasing costs. It is shown that the principal has to provide a greater sharing pool when moral hazard problems exist. Agency cost, defined as the deviation from Pareto-optimal risk sharing, can be considered as the expected value of getting a perfect monitoring mechanism. In both first-best and second-best cases, the profit-sharing scheme is preferred to fixed salary. It is shown that when the attainable materials quality can be further improved, or the marginal probability of getting high-quality material is increased, the principal should offer a higher sharing pool. Diversification is preferred by the principal since the enhanced diversification forces the managers to bear more risk.

This setting assumes binary levels of materials quality, high or low. The procurement manager's effort can only increase the possibility of getting high-quality materials. One possible extension is that the procurement manager's effort can also increase the materials' quality by working with the vendors closely which is the Just-in-Time procurement concept. Another extension is that the production manager's effort or the materials' quality can decrease overall production costs and lessen the volatility of production. In practice, variance reduction is more important than the control of the mean in the production process in many industries, which is the idea of robust quality proposed by Taguchi and Clausing (1990).

#### Degree of Supervision and Moral Hazard in Hierarchical Control

The necessity of production supervision and the payoff of establishing an expanded hierarchical structure are the major concerns in this

chapter. The production workers' moral hazard problem and the principal's limited supervision effort and span of control explain the desirability of setting up an additional layer in the hierarchy by hiring supervising managers. Five models consisting of different structures are compared to show that it is indeed to the principal's benefit to expand the hierarchy levels and organization size.

Possible extensions can be made by considering the following factors. The supervising manager's utility structure may differ from the worker's. Some managers have broader objectives. One case is that the number of the manager's subordinates, i.e. his span of control, might have positive (or negative) effect on his utility. In this chapter, the managers' compensations are the only costs of setting up supervisory positions. Adding one more layer, however, might make communication and control more difficult. Communication at higher levels usually differs from communication at the lower levels (Graicunas, 1937; Ouchi and Dowling, 1974; Keren and Levhar, 1979). The worker's action might be influenced not only by the monitoring scheme but also by (1) the social norms wherein the behavior is not pulled by the prospect of future rewards, but is pushed from behind by quasi-inertial forces (Gambetta, 1987; Elster, 1989), or by (2) group effects wherein the presence of others enhances the emission of dominant behavior (Zajonc, 1965; Zajonc and Sales 1966). It is also assumed in this chapter that the superiors in the hierarchy are fully knowledgeable of managing production. In practice, first-line workers might have better information. Different information structures can be considered for further research.

In conclusion, the above models provide several insights into how to resolve conflicts of interest within organizations by carefully designed compensation schemes. Unlike conventional contracts, the pattern and design of incentive systems depend on exogenous parameters such as the agent's risk and effort attitudes, the variance of environmental uncertainty, and the limitation of the principal's capability. This study also shows that the organization can be more profitable by improving several exogenous variables or by adjusting its structure. Using these findings to guide and influence agency relationships, realization of a more efficient, manageable and productive organization can be attained.



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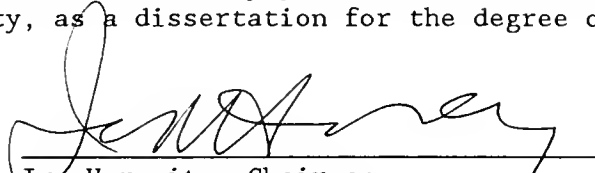
## BIOGRAPHICAL SKETCH

Yeong-Ling (Helio) Yang was born on August 8, 1960, in Taiwan, Republic of China. She earned her bachelor's degree in industrial management sciences with honors from National Cheng Kung University, Taiwan, in 1982. In August 1984, she received the Master of Science degree in industrial and systems engineering from University of Florida. While attending University of Florida from 1984 to 1988, she worked as a graduate assistant in the Department of Industrial and Systems Engineering and the Department of Decision and Information Sciences.

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